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★**Jordan operator algebras.**

From the preface: "Jordan algebras were first studied by Jordan, von Neumann and Wigner in the mid-1930s with the aim of being a suitable setting for axiomatic quantum mechanics. Later on, however, the subject became mainly a branch of algebra, and it was not until the mid-1960s that Jordan algebras were systematically studied from the point of view of functional analysis. From then on a theory has been developed which closely resembles that of C^* and von Neumann algebras, and which is concerned with the infinite-dimensional analogues of the original algebras of Jordan, von Neumann and Wigner."

After a chapter of preliminaries in functional analysis, the monograph develops the algebraic theory of Jordan algebras up to the Jordan, von Neumann and Wigner theorem characterizing finite-dimensional simple formally real Jordan algebras. The rest of the book studies JB algebras and their weakly closed analogues, the JBW algebras, and culminates in the (1978 and 1979) structure theorems for JB and JBW algebras due to Alfsen, Shultz and Stormer: A JB algebra is the Jordan Banach algebra analogue of an abstractly given C^*-algebra (abstractly given C^*-algebras used to be called B^*-algebras). A JC algebra is any norm closed Jordan subalgebra of selfadjoint bounded linear operators on a Hilbert space. A JB algebra B is said to be purely exceptional if there is no nonzero homomorphism of B into a JC-algebra. The authors prove that a JB algebra B is purely exceptional if and only if every factor representation of B is onto the 3-by-3 Hermitian matrices over the Cayley numbers. Also, any JB algebra B contains a unique purely exceptional ideal J such that B/J is a JC algebra. Any JBW algebra can be uniquely decomposed as a direct sum of a weakly closed JC algebra and a purely exceptional algebra.

The main interest in Jordan operator algebras has been their close relationship to order-theoretic questions in C^*-algebras. However, their main current interest is probably due to the relationship between JB algebras and bounded symmetric domains in several (or infinitely many) complex variables. This is mentioned very briefly in the monograph, but for the details the reader should consult the recent book by H. Upmeier [Symmetric Banach manifolds and Jordan C^*-algebras, North-Holland, Amsterdam, 1985]. Also, the characterization of the state spaces of Jordan algebras and C^*-algebras [E. M. Alfsen , Hanche-Olsen and F. W. Shultz, Acta Math. 144 (1980), no. 3-4, 267–305; MR0573454 (81g:46074)] is not covered in the monograph.

Reviewed by *J. W. Bunce*