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them is based on the heat equation method and has led to further generalizations.

The book of J. Roe gives an introduction to the circle of ideas surrounding the heat equation proof of the Atiyah–Singer index theorem. The analysis is worked out in the context of Dirac operators on Clifford bundles that gives classical elliptic operators arising from geometry.

Briefly, the heat equation proof of the index theorem proceeds as follows. On each S_j choose an Hermitian metric and use these metrics to construct the adjoints d_j^* of the d_j. Consider the Laplacians $\Delta_j = d_j^* d_j + d_{j-1} d_{j-1}^*$ and define the heat operators $e^{-t\Delta_j}$.

The first step is to express $\operatorname{Ind}(S, d)$ by means of these heat operators: for all $t > 0$

$$\operatorname{Ind}(S, d) = \sum_{j=0}^{k} (-1)^j \operatorname{Tr}(e^{-t\Delta_j}).$$

In the second place, we use the asymptotic expansions of the Schwartz kernels of these operators to establish the behaviour of $\operatorname{Tr}(e^{-t\Delta_j})$ as $t \to 0$, and obtain the index formula.

In the Roe's book this method is also applied to the spectral geometry problem, the Atiyah–Bott–Lefschetz formula, the analytic Witten approach to Morse inequalities, and the Atiyah index theorem for coverings.

Roe's book is very carefully written and all the differential geometry (including Hodge theory), functional analysis, and PDE (including the analysis of Dirac operators) needed is developed in the text. Some exercises are provided.

This monograph is a welcome addition to the list of books for people who want to learn about modern analysis and geometry.

Institute of Mathematics,
Novosibirsk, U.S.S.R.

N. S. DAIRBEKOV

S. Albeverio, F. Gesztesy, R. Høegh-Krohn, and H. Holden: *Solvable Models in Quantum Mechanics*, Springer-Verlag, Berlin, 1988, 452pp., DM 158.

This monograph is devoted to a detailed investigation of a class of models of quantum mechanics given by a Schrödinger Hamiltonian with potential supported on a discrete finite or infinite) set of points (=sources). These interactions are known under various names like such as, e.g., 'point interactions', 'zero-range potentials', 'delta interactions', 'Fermi pseudopotentials', and 'contact interactions'.

The main basic quantum mechanical systems which are discussed in the monograph are heuristically given by one particle many center Hamiltonians of the form

$$H = -\Delta + \sum_{y \in Y} \lambda_y \delta_y(\cdot),$$

where Δ denotes the Laplacian in $L^2(R^d)$, $d = 1, 2, 3$ is the dimension of the configuration space, Y is a discrete (finite or countably infinite) subset of R^d, λ_y is a coupling, and δ_y is the Dirac δ-function at y.

Such point interaction models are 'solvable' in the sense that their resolvents and associated mathematical and physical quantities, such as the spectrum, the corresponding eigenfunctions, resonances, and scattering quantities, can be determined explicitly in terms of the interaction strengths and the location of the sources.

We would like to mention that this monograph does not discuss the class of solvable quantum mechanical models, the methods of solutions of which are based on a group-theoretical approach and quantum inverse scattering method and which are closely connected with classically completely integrable systems. About these models, see, e.g., [1, 2] and references therein.

Solvable models considered in the monograph play an important role in the mathematical modeling of many natural phenomena. These models are used in solid state physics, atomic and nuclear physics, and electromagnetism. They make it possible to model essential features of the phenomena and then, using suitable methods, to handle more complicated and realistic situations, see, e.g., [3]. In addition, we also remark that a more detailed investigation of multiparticle point interactions is given in a monograph by Gaudin [4].

Heuristical and physical considerations of such models were begun long ago. Historically, the first paper on models of such a type was that by Kronig and Penney, in 1931, who treated the Kronig–Penney model of a crystal. A few years later, Bethe and Peierls and Thomas discussed models with a point interaction at one point in a three-dimensional space in order to describe the interaction of a nonrelativistic quantum mechanical particle interacting via a 'very short range' potential with a fixed source and, in this case, it was shown that a renormalization of the coupling constant is necessary.

The first rigorous mathematical works that considered the point interactions (for $d = 3$) were papers by Berezin, Faddeev, and Minlos in 1961–1962. They used Krein's theory of self-adjoint extensions and took into account the fact that on the space $C_0^\infty(R^d \backslash Y)$, H should coincide with $-\Delta$.

The main purpose of this monograph is to present, in a systematic way, the mathematical approach developed in recent years, to models with point interactions.

The subject is divided into three parts corresponding to point interactions with one center (Part I), finitely many centers (Part II), and infinitely many centers (Part III) in dimensions $d = 1, 2, 3$. The consideration is given with the help of Krein's theory of self-adjoint extensions and approximations of point interactions by interactions with short-range radii tending to zero and some other approximations.

Another mathematical approach to point interactions is provided by nonstandard analysis. This approach is briefly considered in Appendix H. For other applications of nonstandard analysis, see, e.g., [5].

In conclusion, it should be noted that the monograph, except the beautiful exposition, contains a huge bibliography. This monograph will be very useful to all specialists who are interested in the point interactions.

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Department of Theoretical Physics,
Novosibirsk, U.S.S.R.

E. P. OSIPOV