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Soliton equations and their algebro-geometric solutions. Volume I: (1+1)-dimensional continuous models. (English)

Cambridge Studies in Advanced Mathematics 79. Cambridge: Cambridge University Press. xii, 505 p. £65.00; $ 95.00 (2003). [ISBN 0-521-75307-4/hbk]

This is the first volume in a series of monographs by the authors devoted to an effective approach to the construction of algebro-geometric solutions of soliton hierarchies. It deals with soliton hierarchies in 1+1 dimensions, based on their hyperelliptic curves of the type $y^2 = \prod_{m=0}^{N}(z - E_m)$ from 2×2 matrix spectral problems and their recursion structures from hereditary symmetry operators. There are five chapters and ten appendices. The book starts with a long introduction, which explains in a descriptive and expositive way the authors' approach to algebro-geometric solutions working for all members in a soliton hierarchy, by considering the Korteweg-de Vries (KdV) hierarchy. The main text is the careful treatment of algebro-geometric solutions for the KdV hierarchy, the combined sine-Gordon and modified KdV (sGmKdV) hierarchy, the Ablowitz-Kaup-Newell-Segur (AKNS) hierarchy, the classical massive Thirring (Th) system and the Camassa-Holm (CH) hierarchy. The representatives in the corresponding hierarchies read

$$\text{KdV:} \qquad u_t + \frac{1}{4}u_{xxx} - \frac{3}{2}uu_x = 0;$$

$$\text{sGmKdV:} \qquad u_{xt} - \sin(u) = 0;$$

$$\text{AKNS:} \qquad p_t + \frac{i}{2}p_{xx} - ip^2q = 0, \ \ q_t - \frac{i}{2}q_{xx} + ipq^2 = 0;$$

$$\text{Th:} \qquad -iu_x + 2v + 2vv^*u = 0, \ \ iu_x^* + 2v^* + 2vv^*u^* = 0,$$

$$-iv_t + 2u + 2uu^*v = 0, \ \ iv_t^* + 2u^* + 2uu^*v^* = 0;$$

$$\text{CH:} \qquad 4u_t - u_{xxt} - 2uu_{xxx} - 4u_xu_{xx} + 24uu_x = 0.$$

The basic techniques adopted come from theories of differential equations, spectral analysis and compact Riemann surfaces; and the key tools are Dubrovin-type equations, trace formulas and canonical meromorphic functions on the underlying hyperelliptic Riemann surfaces. All chapters of the book, each containing one hierarchy, are intended to be essentially self-contained, and thus can be read independently of each other. Though the first chapter has a more theoretical orientation, similar arguments are made for constructing algebro-geometric solutions of different soliton hierarchies in different chapters. References for sections are deferred to detailed notes at the end of the corresponding chapter. In addition to comprehensive bibliographical documentation of the material in the main text, these notes also provide numerous additional comments and results. Succinctly written appendices, supplying summaries of subjects of interest on their own, aim to guarantee a fairly self-contained presentation accessible at an advanced graduate level. The book contains many fundamental ideas, interpretations and

techniques in the field of soliton theory and integrable systems. It is a valuable source of soliton equations for graduate students and researchers, especially for ones who like to study or enter soliton theory or to know beautiful interplays between geometry and algebra in differential equations.

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Keywords : soliton equations; Algebro-geometric solutions; hyperelliptic curves; Dubrovin-type equations; trace formulas

Classification :

  ∗37K40 Soliton theory, asymptotic behavior of solutions

  37K20 Relations with algebraic geometry, etc.

  14H70 Automorphisms of curves

  35Q51 Solitons

  37K10 Completely integrable systems etc.

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