

MUTATING LOOPS AND 2-CYCLES IN 2-CY TRIANGULATED CATEGORIES

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ABSTRACT. We derive a method for mutating quivers of 2-CY tilted algebras that have loops and 2-cycles, under certain specific conditions. Further, we give the classification of the 2-CY tilted algebras coming from standard algebraic 2-CY triangulated categories with a finite number of indecomposables. These algebras satisfy the setup for our method of mutation.

1. INTRODUCTION AND RESULTS

Mutation has played an important role in representation theory during the recent years, especially in tilting and cluster-tilting theory. For instance, let \mathcal{H} be a hereditary abelian k -category over a field k , with finite dimensional Hom-spaces and Ext^1 -spaces having a tilting object T . If \mathcal{H} has no nonzero projective (or nonzero injective) objects, we know that every almost complete tilting object (that is, a tilting object where one indecomposable summand is removed) has exactly two complements (see [H2, HU, BOW2]). Thus we can always replace any indecomposable summand of T , to obtain a new tilting object T' . This procedure is called (tilting) mutation in \mathcal{H} . Unfortunately, this procedure does not work in general if \mathcal{H} has nonzero projectives.

Fortunately, there exists a generalization of tilting theory, where mutation is always possible: *cluster-tilting theory*. The approach goes as follows: Trying to model cluster algebras from a categorical point of view, the authors in [BMRRT] introduced the cluster category \mathcal{C}_H for a hereditary algebra H (see also [CCS] for the \mathbb{A}_n case), and more generally for a hereditary category \mathcal{H} with a tilting object. The cluster category comes equipped with a class of objects called *cluster-tilting objects*. There is a mutation of cluster-tilting objects in cluster categories, and by [IY] more generally in Hom-finite triangulated 2-Calabi-Yau categories (2-CY for short).

Associated to a tilting object T in \mathcal{H} is the endomorphism algebra $\text{End}_{\mathcal{H}}(T)$, called *quasi-tilted algebra* ([HRS]). The mutation of tilting objects induces a mutation of quasi-tilted algebras (see [H1]). Similarly, associated to a cluster-tilting object T in \mathcal{C} is the endomorphism algebra $\text{End}_{\mathcal{C}}(T)$, called *cluster-tilted algebra* ([BMR2]). The mutation of cluster-tilting objects also induces a mutation of cluster-tilted algebras, and further a mutation of their quivers (see [BIRSm]). It coincides (see [BMR3, BIRSc]) with the quiver mutation rule given by S. Fomin and A. Zelevinsky in [FZ] in their theory of cluster algebras.

The main limitation of this quiver mutation rule is, that one only mutates at vertices not lying in loops or 2-cycles. At the categorical level, there is no such restriction. Therefore one would expect that there is a way to generalize the quiver mutation rule to vertices lying on loops and 2-cycles. The aim of this paper is to give a procedure for mutating quivers of 2-CY tilted algebras (the endomorphism

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rings of cluster-tilting objects in 2-CY triangulated categories) at vertices with loops and 2-cycles, under some special conditions.

The setup is the following: We assume to have a Galois covering $\pi: \mathcal{T} \rightarrow \overline{\mathcal{T}}$ of algebraic 2-CY triangulated categories, where no cluster-tilting object in \mathcal{T} has loops or 2-cycles. For a (basic) cluster-tilting object $\overline{\tau}T$ in $\overline{\mathcal{T}}$ having loops and/or 2-cycles, we denote by τT its lift in \mathcal{T} . Then τT is a cluster-tilting object in \mathcal{T} (see Proposition 4.1).

First, we develop a procedure to replace the fibre of an indecomposable summand of $\overline{\tau}T$ in τT , in order to obtain a new cluster-tilting object $\tau T'$ in \mathcal{T} . Then we show that $\pi(\tau T')$ coincides with replacing the corresponding indecomposable summand of $\overline{\tau}T$ in $\overline{\mathcal{T}}$.

Second, we develop a corresponding procedure at the level of quivers. That is, if we denote by \overline{Q} (resp. Q) the quiver of the 2-CY tilted algebra associated to $\overline{\tau}T$ (resp. τT), we give a method to mutate at any given vertex v of \overline{Q} by mutating its cover Q at the fibre $\pi^{-1}(v)$, where $\pi: Q \rightarrow \overline{Q}$ also denotes the induced covering morphism of quivers.

Finally, we give the classification of the 2-CY tilted algebras of finite type. We show that the algebras in this class satisfy the setup for our method of mutation, and organize them according to their mutation classes.

The constructions above rely heavily on a reduction technique by [IY], and a generalized mutation rule for algebraic 2-CY triangulated categories by [P].

The paper is organized as follows.

In Section 2 we define the notation we use and recall some basic results on mutation of quivers, quivers with potentials, cluster categories, coverings, Palu's generalized mutation rule for algebraic 2-CY triangulated categories, and Iyama-Yoshino's reduction technique.

In Section 3, we develop the theory to replace, at the same time, several summands of a cluster-tilting object in an algebraic 2-CY triangulated category. Then we derive our rule to mutate at (minimal) oriented cycles of 2-CY tilted algebras (due to the length of the calculations, they are postponed to Appendix 6). Furthermore, we prove that mutating at cycles is equivalent to a sequence of FZ-mutations.

In Section 4 we present the method to mutate quivers of 2-CY tilted algebras having loops and/or 2-cycles.

Finally, in Section 5 we present the classification of the 2-CY tilted algebras coming from standard algebraic 2-CY triangulated categories with a finite number of indecomposables. By using our mutation procedure, we are able to mutate at any vertex in the quivers of these algebras.

2. BACKGROUND

2.1. Conventions. Fix k an algebraically closed field. When we say that \mathcal{C} is a category, we always assume that \mathcal{C} is k -linear additive with finite dimensional morphism spaces.

Let \mathcal{C} be a category. We denote by $\mathcal{C}(X, Y)$ or by (X, Y) the set of morphisms from X to Y in \mathcal{C} . An *ideal* I of \mathcal{C} is an additive subgroup $I(X, Y)$ of $\mathcal{C}(X, Y)$ such that $fgh \in I(W, Z)$ whenever $f \in \mathcal{C}(W, X)$, $g \in I(X, Y)$, and $h \in \mathcal{C}(Y, Z)$. For an ideal I of \mathcal{C} , we write \mathcal{C}/I for the category whose objects are the objects of \mathcal{C} and whose morphisms are given by $\mathcal{C}/I(X, Y) = \mathcal{C}(X, Y)/I(X, Y)$ for $X, Y \in \mathcal{C}/I$.

When we say that \mathcal{D} is a subcategory of \mathcal{C} , we always mean that \mathcal{D} is a full subcategory that is closed under isomorphisms, direct sums, and direct summands. We denote by $[\mathcal{D}]$ the ideal of \mathcal{C} consisting of morphisms that factor through objects in \mathcal{D} . Thus we can form the category $\mathcal{C}/[\mathcal{D}]$.

A morphism f is said to be *right minimal* if it does not have a summand of the form $X_0 \rightarrow 0$ as a complex, for a nonzero object $X_0 \in \mathcal{C}$. For a subcategory \mathcal{D} of \mathcal{C} , a morphism f is called a *right \mathcal{D} -approximation* of $Y \in \mathcal{C}$ if $X \in \mathcal{D}$ and

$$\mathcal{C}(-, X) \xrightarrow{(-, f)} \mathcal{C}(-, Y) \rightarrow 0,$$

is an exact sequence of functors on \mathcal{D} . We say that a right \mathcal{D} -approximation is *minimal* if it is right minimal. A subcategory \mathcal{D} is called a *contravariantly finite subcategory* of \mathcal{C} if any $Y \in \mathcal{C}$ has a right \mathcal{D} -approximation. Dually one defines a *left \mathcal{D} -approximation* and a *covariantly finite subcategory*. A contravariantly and covariantly finite subcategory is said to be *functorially finite*.

Let \mathcal{X} be a subcategory of a category \mathcal{T} . Define \mathcal{X}^\perp to be the subcategory of all $T \in \mathcal{T}$ such that $(\mathcal{X}, T) = 0$. Dually, ${}^\perp\mathcal{X} = \{T \in \mathcal{T} \mid (T, \mathcal{X}) = 0\}$.

A k -linear autofunctor $\nu: \mathcal{T} \rightarrow \mathcal{T}$ of a triangulated category \mathcal{T} is called a *Serre functor* of \mathcal{T} if there is a functorial isomorphism $(X, Y) \simeq D(Y, \nu X)$ for any $X, Y \in \mathcal{T}$, where D denotes the usual k -duality. If \mathcal{T} has a Serre functor, then it is unique (up to natural isomorphism). We say that \mathcal{T} is *n -Calabi-Yau* (n -CY for short) for an integer $n \in \mathbb{Z}$ if $\nu = [n]$.

A triangulated category is *algebraic* if it is triangle equivalent to the stable category of a Frobenius exact category. All triangulated categories occurring throughout this paper are assumed to be algebraic.

2.2. Quiver mutation. Let Q be a finite quiver with vertices $1, \dots, n$ having no loops or 2-cycles. Let $q_{i,j}$ denote the number of arrows from i to j minus the number of arrows from j to i in Q . The terms $q_{i,j}$ can be collected in a matrix called the *skew-symmetric matrix* associated to Q . Fomin-Zelevinsky [FZ] defined a mutation rule that applies to this setting. For a vertex ℓ , we obtain the new quiver $\mu_\ell(Q)$ as follows: The skew-symmetric matrix $(q'_{i,j})$ associated to $\mu_\ell(Q)$ is given by

$$q'_{i,j} = \begin{cases} -q_{i,j} & \text{if } i = \ell \text{ or } j = \ell, \\ q_{i,j} + \frac{|q_{i,\ell}q_{\ell,j} + q_{i,\ell}q_{\ell,j}|}{2} & \text{otherwise.} \end{cases}$$

We say that Q and $\mu_\ell(Q)$ are mutations of one another. Observe that $\mu_\ell(\mu_\ell(Q)) \simeq Q$. The collection of all quivers which are iterated mutations of Q is called the *mutation class* of Q .

2.3. Quivers with potentials. We follow [DWZ]. Let Q be a quiver (possibly with loops and 2-cycles). Denote by Q_0 its set of vertices (or, equivalently, the paths of length zero) and by Q_i the paths of length i in Q , where i is a positive integer. Let kQ_i be the k -vector space with basis Q_i and denote by kQ_i^c the subspace of kQ_i generated by all the cycles. The complete path algebra of Q is then defined as

$$\widehat{kQ} = \prod_{i \geq 0} kQ_i,$$

that is, the completion of the path algebra kQ with respect to the ideal generated by all the arrows. For an integer $m \geq 2$ let

$$\text{Pot}_m(kQ) = \prod_{i \geq m} kQ_i^c.$$

An element $\mathcal{P} \in \text{Pot}_2(kQ)$ is called a *potential* on Q . It is called *reduced* if $\mathcal{P} \in \text{Pot}_3(kQ)$. We say that two potentials are *cyclically equivalent* if their difference is in the closure of the span of all elements of the form $p_1 \cdots p_d - p_2 \cdots p_d p_1$ where $p_1 \cdots p_d$ is a cyclic path.

For an arrow p of Q we define $\partial_p: \text{Pot}_2(kQ) \rightarrow \widehat{kQ}$ as the continuous k -linear map taking a cycle c to the sum $\sum_{c=xpy}yx$ taken over all decompositions of the cycle c (where x or y are possibly paths of length zero). It is clear that two cyclically equivalent paths have the same image under ∂_p . We call ∂_p the *cyclic derivative with respect to p* .

Let \mathcal{P} be a potential on Q such that no two cyclically equivalent cyclic paths appear in \mathcal{P} . We call the pair (Q, \mathcal{P}) a *quiver with potential* (or QP for short). Associated to a QP we have a *Jacobian algebra* defined as

$$\mathcal{J}(Q, \mathcal{P}) = \widehat{kQ}/I(\mathcal{P}),$$

where $I(\mathcal{P})$ is the closure of the ideal generated by all $\partial_p \mathcal{P}$ where p runs over all arrows of Q .

For further details we refer the reader to [DWZ].

2.4. Cluster categories. The cluster category \mathcal{C} of a hereditary category \mathcal{H} was introduced in [BMRRT]. It is defined as the orbit category \mathcal{D}/F , where F is the automorphism $\tau^{-1}[1]$ with τ the Auslander Reiten translation, and $[1]$ the shift functor of $\mathcal{D} := D^b(\mathcal{H})$. This gives an algebraic triangulated ([K]) and Krull-Schmidt ([BMRRT]) category, which is an important tool for studying the tilting theory of \mathcal{H} . It also models the combinatorics of the acyclic cluster algebras introduced by Fomin and Zelevinsky (see [FZ]) in a natural way.

The objects of \mathcal{C} are the same as in \mathcal{D} . Given objects X and Y in \mathcal{D} , the space of morphisms $\text{Hom}_{\mathcal{C}}(X, Y)$ is defined as $\coprod_{i \in \mathbb{Z}} \text{Hom}_{\mathcal{D}}(F^i X, Y)$. The morphisms in \mathcal{C} are thus induced by morphisms and extensions in \mathcal{H} .

The category comes with a set of distinguished objects, the *cluster-tilting objects*. These are maximal rigid objects (maximal with respect to the number of non-isomorphic indecomposable summands). If T is a cluster-tilting object in \mathcal{C} , then $\text{End}_{\mathcal{C}}(T)$ is called a *cluster-tilted algebra* ([BMR2] – in that paper the authors actually chose to look at the opposite algebra of $\text{End}_{\mathcal{C}}(T)$).

We shall be particularly interested in the cluster-tilted algebras of finite type. These were characterized by [BMR2] as follows: Let $B = \text{End}_{\mathcal{C}}(T)$ be a cluster-tilted algebra with $\mathcal{C} = \mathcal{C}_H$ the cluster category of some hereditary algebra H , and T a tilting object in \mathcal{C} . We then have that B is of finite representation type if and only if H is of finite representation type. In this case H is the path algebra of a Dynkin quiver Q , and the underlying graph Δ of Q is one of $\{\mathbb{A}_n, \mathbb{D}_m, \mathbb{E}_6, \mathbb{E}_7, \mathbb{E}_8\}$ for $n \geq 1$ and $m \geq 4$. In this case, we say that \mathcal{C} and B are of type Δ .

Finite type cluster-tilted algebras are (up to Morita equivalence) determined uniquely by their quiver by [BMR1]. Furthermore, their relations are determined by a potential, which is given by the sum of all minimal cycles of the corresponding

quiver ([BMR1])¹. Recall that a cycle is *minimal* if the subquiver generated by the cycle contains only arrows of the cycle and every vertex appears only once.

2.5. Coverings. We follow [G]. However our notation varies slightly from [G], since our categories have finite direct sums and isomorphic objects which are not equal.

A (k -linear) functor $F: \mathcal{C} \rightarrow \mathcal{D}$ is called a *covering functor* if the induced maps

$$\bigoplus \mathcal{C}(X, Y) \rightarrow \mathcal{D}(FX, A) \text{ and } \bigoplus \mathcal{C}(Y, X) \rightarrow \mathcal{D}(A, FX)$$

are bijective for all $X \in \mathcal{C}$ and indecomposable $A \in \mathcal{D}$, where in both cases the sum runs over all isomorphism classes of objects Y such that $FY \simeq A$.

Let \mathcal{C} be a Hom-finite Krull-Schmidt category and G a group of (k -linear) automorphisms of \mathcal{C} . Assume that the action of G on \mathcal{C} is *free* (that is $gX \not\simeq X$ for each $X \in \mathcal{C}$ and $1 \neq g \in G$) and *locally bounded* (for each pair X, Y there are only finitely many $g \in G$ such that $(X, gY) \simeq (g^{-1}X, Y) \neq 0$). Then we have the following proposition.

Proposition 2.1 ([G, 3.1]). *The quotient \mathcal{C}/G exists in the category of all Hom-finite k -categories, and the canonical projection $\pi: \mathcal{C} \rightarrow \mathcal{C}/G$ is a covering functor.*

Suppose that we have a covering functor $F: \mathcal{C} \rightarrow \mathcal{D}$ such that $Fg = F$ for all $g \in G$. Such a functor induces an isomorphism $\mathcal{C}/G \simeq \mathcal{D}$ if and only if F is surjective on the objects and G acts transitively on the fiber $F^{-1}(A)$ for each object $A \in \mathcal{D}$. If this is the case, we call F a *Galois covering*.

For further details we refer the reader to [BG, G].

2.6. Palu's formula. In this subsection we recall the generalized mutation rule for algebraic 2-CY triangulated categories from [P]. This formula is one of the key ingredients in our method for mutating at loops and 2-cycles. We state the result in the following setting: Let T be a cluster-tilting object in the algebraic 2-CY triangulated category \mathcal{T} . Delete the loops and oriented 2-cycles from the quiver of (T, T) , and denote the remaining quiver by Q . Let $M = (m_{i,j})$ be the skew-symmetric matrix (see Subsection 2.2) associated to Q . Furthermore, let $T' \in \mathcal{T}$ be another cluster-tilting object, and define the quiver Q' and the skew-symmetric matrix M' in a likewise manner.

We now approximate T with respect to T' . We can write $T = \bigoplus_j T_j$ with T_j indecomposable, and decompose T' similarly. We have that for each j there is a triangle (see [BMRRT, IY, KR]) of the form

$$T_j[-1] \rightarrow \bigoplus_i \beta_{i,j} T'_j \rightarrow \bigoplus_i \alpha_{i,j} T'_i \rightarrow T_j$$

and we define the matrix $S = (s_{i,j})$ by setting $s_{i,j} = \alpha_{i,j} - \beta_{i,j}$. Then by [P, Theorem 12 a)] we have that

$$(2.1) \quad M' = SMS^t.$$

¹The result in [BMR1] used a different equivalent description.

2.7. Iyama-Yoshino's reduction. This subsection collects the results we are going to use from [IY], but restricted to the following setting: Let \mathcal{T} be a 2-CY triangulated category. Fix a functorially finite subcategory \mathcal{D} of \mathcal{T} satisfying $(\mathcal{D}, \mathcal{D}[1]) = 0$. Using \mathcal{D} , we construct the subcategory $\mathcal{Z} := {}^{\perp}\mathcal{D}[1]$ and the subfactor category $\mathcal{U} := \mathcal{Z}/[\mathcal{D}]$ of \mathcal{T} . Then the category \mathcal{U} is triangulated ([IY, Theorem 4.2]) and 2-CY ([IY, Theorem 4.9]). We now describe the shift functor $\langle 1 \rangle$ and the standard triangles in \mathcal{U} .

For any $X \in \mathcal{Z}$, fix a triangle

$$X \xrightarrow{\alpha_X} D_X \xrightarrow{\beta_X} X\langle 1 \rangle \xrightarrow{\gamma_X} X[1]$$

where α_X is a left \mathcal{D} -approximation, and define $X\langle 1 \rangle$ by this (then β_X automatically is a right \mathcal{D} -approximation). The action of $\langle 1 \rangle$ on morphisms uses commutative diagrams of triangles like the one above (see [IY, Definition 2.5]).

Let $X \xrightarrow{a} Y \xrightarrow{b} Z \xrightarrow{c} X[1]$ be a triangle in \mathcal{T} with $X, Y, Z \in \mathcal{Z}$. Since $\mathcal{T}(Z[-1], D_X) = 0$ holds, there is a commutative diagram

$$\begin{array}{ccccccc} X & \xrightarrow{a} & Y & \xrightarrow{b} & Z & \xrightarrow{c} & X[1] \\ \downarrow 1_X & & \downarrow & & \downarrow d & & \downarrow 1_{X[1]} \\ X & \xrightarrow{\alpha_X} & D_X & \xrightarrow{\beta_X} & X\langle 1 \rangle & \xrightarrow{\gamma_X} & X[1] \end{array}$$

Now the standard triangles in \mathcal{U} are the complexes of the form $X \xrightarrow{\bar{a}} Y \xrightarrow{\bar{b}} Z \xrightarrow{\bar{c}} X\langle 1 \rangle$.

3. MUTATING AT ORIENTED CYCLES

In this section we develop the theory to mutate a cluster-tilting object in several summands (satisfying certain specific conditions) at the same time, in an algebraic 2-CY triangulated category. To achieve this, we rely heavily on Palu's formula from Section 2.6 and the Iyama-Yoshino construction recalled in Section 2.7.

3.1. Exchanging several summands. Throughout this section let \mathcal{T} be an algebraic 2-CY triangulated category, and fix $T = T_m \oplus T_f$ a cluster-tilting object in \mathcal{T} , where neither of the summands T_m or T_f is necessarily indecomposable. Let $\mathcal{D} := \text{add } T_f$. Then clearly $(\mathcal{D}, \mathcal{D}[1]) = 0$. Define \mathcal{Z} to be the subcategory ${}^{\perp}\mathcal{D}[1]$ and \mathcal{U} the 2-CY subfactor category $\mathcal{Z}/[\mathcal{D}]$ of \mathcal{T} . By [IY, Theorem 4.9] we have a one-to-one correspondence between cluster-tilting objects in \mathcal{T} having T_f as a summand and cluster-tilting objects in \mathcal{U} .

The main purpose of this section is to mutate the summand T_m of T , and leave the remaining part fixed, i.e. we want to replace T_m by T'_m in such a way that $T' = T'_m \oplus T_f$ is again a cluster-tilting object in \mathcal{T} .

In order to do this, we follow the construction explained in Section 2.7. Consider the following triangle in \mathcal{T}

$$T_m \xrightarrow{a} D \xrightarrow{b} T_m\langle 1 \rangle \rightarrow T_m[1]$$

where a (resp. b) is a minimal left (resp. right) \mathcal{D} -approximation and $\langle 1 \rangle$ is the shift functor in \mathcal{U} . The object $T_m\langle 1 \rangle \oplus T_f$ is cluster-tilting in \mathcal{T} , since $T_m\langle 1 \rangle$ is cluster-tilting in \mathcal{U} .

One could also make the dual construction, by using the following triangle

$$T_m[-1] \rightarrow T_m\langle -1 \rangle \xrightarrow{b'} D' \xrightarrow{a'} T_m$$

where a' (resp. b') is a minimal right (resp. left) \mathcal{D} -approximation.

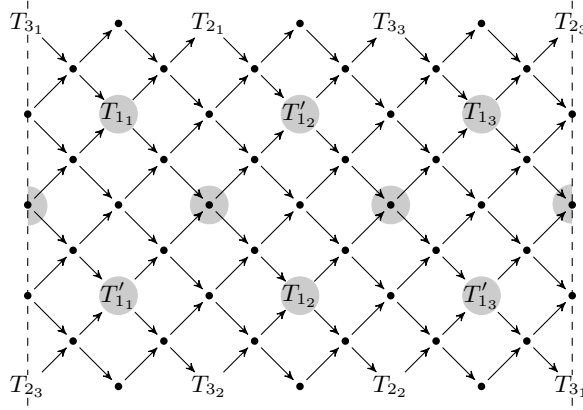
We want to construct the replacement T'_m symmetrically, hence we require $T'_m = T_m\langle 1 \rangle$ and $T'_m = T_m\langle -1 \rangle$. Thus we need $T_m\langle 1 \rangle \simeq T_m\langle -1 \rangle$.

Construction 3.1. Using the notation as above, assume that $T_m \simeq T_m\langle 2 \rangle$ in \mathcal{U} . Then we define the mutation at the summand T_m of the cluster-tilting object T in \mathcal{T} to be

$$\mu_{T_m}(T) = T_m\langle 1 \rangle \oplus T_f$$

Remark 3.2. The assumption in the construction above is equivalent to requiring that the algebra $\mathcal{T}(T_m, T_m)/[\mathcal{D}]$ is self-injective (see [R]).

Example 3.3. Let \mathcal{T} be the cluster category of type \mathbb{A}_9 (This category will be called $\mathcal{A}_{9,1}$ in Definition 5.3), and $T = \bigoplus_{i,j} T_{i,j}$ the cluster-tilting object in \mathcal{T} depicted in the figure below. We want to mutate T at $T_m = \bigoplus_j T_{1,j}$. Observe that the hypothesis of Construction 3.1 is satisfied, and we obtain $T_m\langle 1 \rangle \simeq \bigoplus_j T'_{1,j}$. This process is depicted in the figure below.



Here, the subfactor category \mathcal{U} of \mathcal{T} , for $T_f = \bigoplus_j (T_{2,j} \oplus T_{3,j})$, is indicated in light grey, and is easily seen to be equivalent to the cluster category of \mathbb{A}_3 .

We now present the setup under which we can mutate at cycles.

Setup 3.4 (for mutation at cycles). Let \mathcal{T} be an algebraic 2-CY triangulated category, and fix $T = T_m \oplus T_f$ a cluster-tilting object in \mathcal{T} , where neither of the summands T_m or T_f is necessarily indecomposable. Let \mathcal{U} be the subfactor category of \mathcal{T} defined by

$$\mathcal{U} := {}^\perp(\text{add } T_f[1]) / [\text{add } T_f].$$

Let $T' = T'_m \oplus T_f$, where the summand $T'_m = T_m\langle 1 \rangle$ and the functor $\langle 1 \rangle$ is the shift in \mathcal{U} . We denote by Q the quiver of (T, T) , by $Q_{m,f}$ its subquiver containing only the arrows corresponding to irreducible maps from T_m to T_f , and similarly for $Q_{f,m}$, $Q_{m,m}$, and $Q_{f,f}$. We denote by Q' the quiver of (T', T') and define $Q'_{a,b}$ correspondingly for all $a, b \in \{m, f\}$. Assume the following:

- (a) No cluster-tilting object in \mathcal{T} has loops or 2-cycles.
- (b) $T_m\langle 2 \rangle = T_m$.
- (c) The quiver $Q_{m,m}$ is a cycle of length $l \geq 3$.

- (d) The relations of (T, T) are given by a potential.
- (e) The algebra $(T_m, T_m)/[T_f]$ is the path algebra of $Q_{m,m}$ with minimal relations given by the paths of length l .

Under this setup, we want to mutate Q at the cycle $Q_{m,m}$. In order to do this, we apply the formula from Equation 2.1. Using the same notation as in Section 2.6 we compute as follows. Write

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} - \begin{pmatrix} A & B \\ C & D \end{pmatrix}^t = \begin{pmatrix} A - A^t & B - C^t \\ C - B^t & D - D^t \end{pmatrix}$$

where $M_{m,m} := A$ (resp. $M_{m,f} := B, M_{f,m} := C, M_{f,f} := D$) is the matrix of the arrows in $Q_{m,m}$ (resp. $Q_{m,f}, Q_{f,m}, Q_{f,f}$).

Observe that A is an $l \times l$ -matrix that looks like

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ & & & & \ddots & 0 \\ 0 & \cdots & & & 0 & 1 \\ 1 & 0 & & \cdots & & 0 \end{pmatrix},$$

a cyclic permutation matrix. To simplify the notation, we denote all the arrows of $Q_{m,m}$ by α , and set α^i to be the composition of i such arrows. We decompose $C = \sum_{i=0}^{l-2} C_i$, where C_i corresponds to the set of arrows

$$C_i = \{ \gamma \in Q_{f,m} \mid \gamma \alpha^{i+1} \text{ is zero or factors through an arrow in } Q_{f,f}, \\ \gamma \alpha^i \text{ is non-zero and does not factor through an arrow in } Q_{f,f} \}.$$

Dually, we decompose $B = \sum_{i=0}^{l-2} B_i$. Now, using the fact that the relations come from a potential, we have the following equalities.

$$(3.1) \quad C_i^t = A^{i+1} B_i \text{ for all } 0 \leq i \leq l-3.$$

Observe that the matrix S (see Subsection 2.6) can be written as

$$S = \begin{pmatrix} -1 & 0 \\ \sum_{i=0}^{l-2} C_i & \sum_{j=0}^i A^j & 1 \end{pmatrix}.$$

In order to get a more symmetric result, we twist S with the permutation matrix $\begin{pmatrix} A^{-1} & 0 \\ 0 & 1 \end{pmatrix}$, and thus we calculate

$$M' = \begin{pmatrix} -A^{-1} & 0 \\ \sum_{i=0}^{l-2} C_i & \sum_{j=0}^i A^j & 1 \end{pmatrix} \begin{pmatrix} A - A^t & B - C^t \\ C - B^t & D - D^t \end{pmatrix} \begin{pmatrix} -A^{-1} & 0 \\ \sum_{i=0}^{l-2} C_i & \sum_{j=0}^i A^j & 1 \end{pmatrix}^t.$$

Then we can prove the following result.

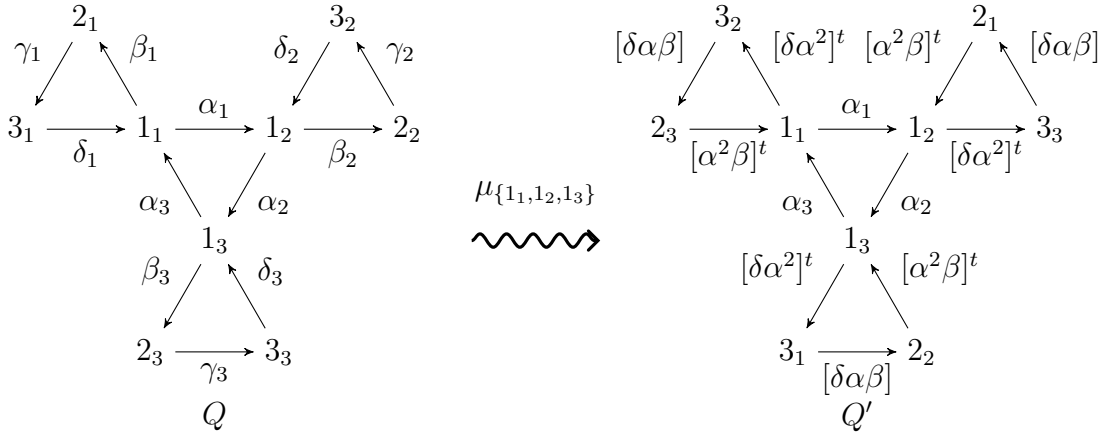
Theorem 3.5 (Mutation rule for cycles). *Suppose that we are in the situation of Setup 3.4. Then mutation at $Q_{m,m}$ has the following effect on the quiver:*

- (a) *Arrows in $Q_{m,m}$ remain unchanged.*

- (b) *Arrows in $Q_{f,m}$: Any arrow γ in C_i , with $0 \leq i \leq l-3$ remains unchanged. For an arrow γ in C_{l-2} , we consider the path $\gamma\alpha^{l-1}$ going from a to b . Then replace γ by an arrow $[\gamma\alpha^{l-1}]^t$ going from b to a .*
- (c) *Arrows in $Q_{m,f}$: Apply the dual process for the arrows in $Q_{m,f}$.*
- (d) *Arrows in $Q_{f,f}$ remain unchanged.*
- (e) *Furthermore, add an arrow $[\gamma\alpha^i\beta]$ for each composition $\gamma\alpha^i\beta$ where $0 \leq i \leq l-2$ and $\gamma \in Q_{f,m}$, $\alpha \in Q_{m,m}$, $\beta \in Q_{m,f}$ such that*
 - (i) *neither $\gamma\alpha^i$ nor $\alpha^i\beta$ is zero or factors through an arrow in $Q_{f,f}$,*
 - (ii) *$\gamma \in C_{l-2}$ or $\beta \in B_{l-2}$, i.e. at least one of γ or β has no extra relations with the cycle of $Q_{m,m}$.*
- (f) *Finally, remove any loops or 2-cycles from the mutated quiver.*

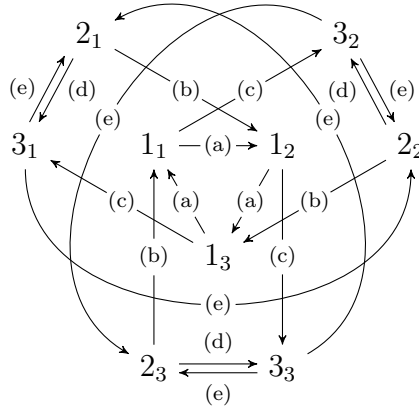
Proof. The calculations are straight-forward but somewhat lengthy. See Appendix 6. \square

Example 3.6. Let T in \mathcal{T} be as in Example 3.3. We depict the quiver Q of (T, T) to the left in the diagram below:



where the potential is given by the sum of all the minimal cycles. It is clear that Setup 3.4 is satisfied. We want to mutate this quiver at the minimal cycle spanned by the set of vertices $\{1_1, 1_2, 1_3\}$. Using the same notation as in the mutation rule, we observe that we have decompositions $C = C_0 + C_1$ and $B = B_0 + B_1$ since the length of the minimal α -cycle is 3. It is easy to see that C_0 and B_0 are empty, $B_1 = \{\beta_i \mid i = 1, 2, 3\}$ and $C_1 = \{\delta_i \mid i = 1, 2, 3\}$. We are ready to apply the rule:

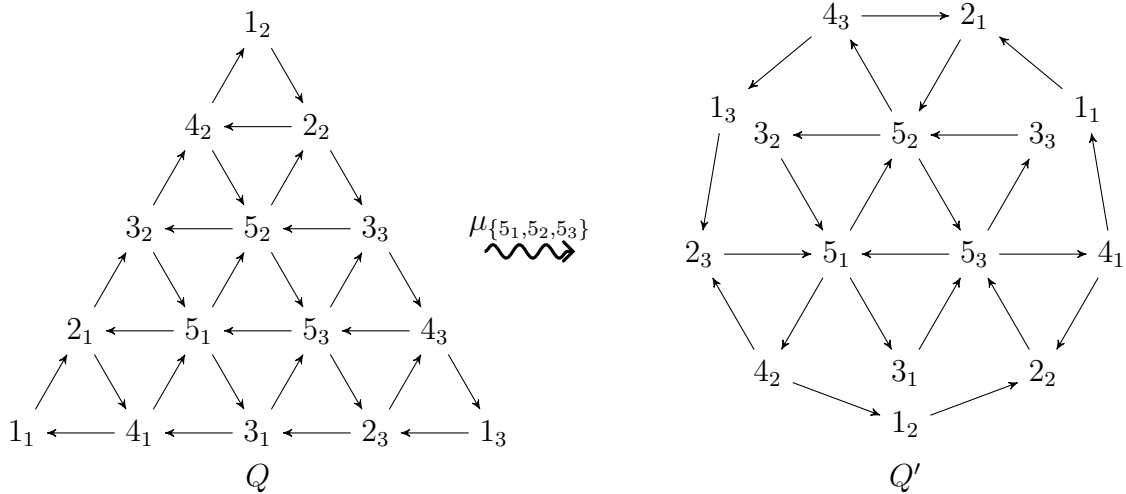
- (a) $Q'_{m,m}$. The α -arrows stay the same.
- (b) $Q'_{m,f}$. For each path $\alpha^2\beta$ we add an arrow $[\alpha^2\beta]^t$ going in the opposite direction.
- (c) $Q'_{f,m}$. For each path $\delta\alpha^2$ we add an arrow $[\delta\alpha^2]^t$ going in the opposite direction.
- (d) $Q'_{f,f}$. The γ -arrows stay the same.
- (e) Furthermore, we add all the compositions $[\delta\beta]$ and $[\delta\alpha\beta]$. Then we end up with a quiver like in the figure shown below. Here we have indicated in which step the arrows have been added.



(f) Now eliminate all loops and 2-cycles.

Then one obtains the quiver Q' to the right in the figure at the beginning of the example, which is the quiver of the endomorphism ring of $\mu_{\{1_1, 1_2, 1_3\}}(T)$ as one can also see from the AR-quiver of \mathcal{T} in Example 3.3.

Example 3.7. Consider the canonical cluster-tilting object T from the stable module category of the preprojective algebra of A_6 . The quiver Q of (T, T) is depicted to the left in the figure below.



where the potential is given by the sum of all the minimal triangles. It is not hard to see that Setup 3.4 is satisfied (to check the no loops and 2-cycles condition see [GLS, BIRSc]). Let $Q_{m,m} = \{5 \rightarrow 5\}$, $Q_{f,m} = \{3 \rightarrow 5, 4 \rightarrow 5\}$, $Q_{m,f} = \{5 \rightarrow 2, 5 \rightarrow 3\}$ and $Q_{f,f}$ the rest. We want to mutate at the minimal cycle of length 3 given by $Q_{m,m}$. Decompose $C = C_0 + C_1$ where $C_0 = \{3 \rightarrow 5\}$ and $C_1 = \{4 \rightarrow 5\}$. Similarly, $B = B_0 + B_1$ where $B_0 = \{5 \rightarrow 3\}$ and $B_1 = \{5 \rightarrow 2\}$. We apply the mutation rule for cycles:

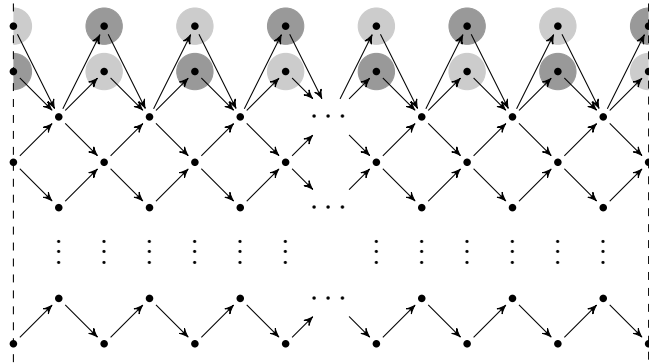
- (a) $5 \rightarrow 5$. These arrows stay the same.
- (b) $5 \rightarrow \{3, 2\}$. For each path $5 \rightarrow 5 \rightarrow 2$ we add an arrow $[2 \rightarrow 5]$. For the paths $5 \rightarrow 5 \rightarrow 3$, we do not add arrows $[3 \rightarrow 5]$ since $5 \rightarrow 5 \rightarrow 3 = 5 \rightarrow 2 \rightarrow 3$, factoring through the arrows $2 \rightarrow 3$ in $Q_{f,f}$.

- (c) $\{4, 3\} \rightarrow 5$. For each path $4 \rightarrow 5 \rightarrow 5$ we add an arrow $[5 \rightarrow 4]$. For the paths $3 \rightarrow 5 \rightarrow 5$, we do not add arrows $[5 \rightarrow 3]$ since $3 \rightarrow 5 \rightarrow 5 = 3 \rightarrow 4 \rightarrow 5$, factoring through the arrows $3 \rightarrow 4$ in $Q_{f,f}$.
- (d) The remaining arrows stay the same.
- (e) Furthermore, we add arrows for all the compositions $[3 \rightarrow 5 \rightarrow 2]$, $[4 \rightarrow 5 \rightarrow 3]$, $[4 \rightarrow 5 \rightarrow 2]$ and $[4 \rightarrow 5 \rightarrow 5 \rightarrow 2]$.
- (f) Now eliminate all loops and 2-cycles.

Then one obtains the quiver Q' to the right in the figure above.

3.2. Mutation rule at cycles vs. FZ-mutation. In both examples above one can check that the result of mutating in a cycle can also be obtained by a sequence of FZ-mutations in the vertices of the cycle (using vertices multiple times, not just every vertex once). In this section we will show that this is no coincidence, but that there always exists a sequence of FZ-mutations that corresponds to our mutation rule at cycles.

Let the cluster-tilting objects $T = T_m \oplus T_f$ and $T' = T_m\langle 1 \rangle \oplus T_f$ in \mathcal{T} , the category \mathcal{U} , and the quivers Q and Q' be as in Setup 3.4. Assume the conditions of Setup 3.4 hold. Using the same notation, we observe that the cluster-tilting object T_m in \mathcal{U} has $\Gamma := (T_m, T_m)/[\text{add } T_f]$ as endomorphism ring, which is a cluster-tilted algebra of type \mathbb{D}_l . It is not hard to find a sequence of mutations taking us from Q_Γ , the quiver of Γ , to a hereditary quiver Q_H . Then, by using [KR, Main Theorem], we see that the 2-CY category \mathcal{U} is triangle equivalent to the cluster category of \mathbb{D}_l . Using the methods developed in [BOW1], we observe that we can choose T_m as indicated in dark gray in the Auslander-Reiten quiver of \mathcal{U} (where l is assumed to be odd) shown below. Here $T_m\langle 1 \rangle$ is indicated in lighter gray.



Since \mathcal{U} is mutation connected, we can always find a sequence of (cluster-tilting) mutations taking us from T_m to $T_m\langle 1 \rangle$. Applying this sequence of mutations in \mathcal{T} , we obtain a sequence of (cluster-tilting) mutations taking us from T to T' . Since \mathcal{T} has no loops or 2-cycles by assumption, cluster-tilting mutation corresponds to FZ-mutation at the level of quivers (see [BIRSc]). Therefore, applying the corresponding sequence of FZ-mutations to the quiver Q , we obtain the quiver Q' . Hence we have proved the following result.

Theorem 3.8. *Let $T = T_m \oplus T_f$ be a cluster tilting object in \mathcal{T} , such that the conditions of Setup 3.4 hold. Let $T' = T_m\langle 1 \rangle \oplus T_f$ be the cluster-tilting object obtained after mutating T at T_m as in Construction 3.1. Denote by Q (resp. Q') the quiver of the 2-CY tilted algebra (T, T) (resp. (T', T')). Let $Q_{m,m}$ denote the quiver*

of (T_m, T_m) . Then there exists a sequence of FZ-mutations taking us from Q to Q' , which correspond to mutating at $Q_{m,m}$ as in Theorem 3.5.

Corollary 3.9. *When mutating in cycles in algebraic 2-CY triangulated categories, the cluster tilting objects remain in the same mutation component.*

In view of Theorem 3.8 and [BIRSm, Proposition 5.1], we have the following improvement of Setup 3.4. Suppose that the 2-CY tilted algebra (T, T) is isomorphic to $\mathcal{J}(Q, \mathcal{P})$, the Jacobian algebra of a QP (Q, \mathcal{P}) (see Section 2.3), where the quiver of (Q, \mathcal{P}) has no loops or 2-cycles. Let $\mu_{v_r} \cdots \mu_{v_1}$ be the sequence of FZ-mutations at the vertices v_1, \dots, v_r taking us from $Q_1 := Q$, to $Q_r := Q'$. Then we can relax condition (a) of Setup 3.4 to the following.

- (a') No 2-cycles start in vertex v_{i+1} in the quiver of $\mu_{v_i} \cdots \mu_{v_1}(Q_1, \mathcal{P}_1)$ for $1 \leq i \leq r - 1$. Here the mutations are of quivers with potential, where $(Q_1, \mathcal{P}_1) := (Q, \mathcal{P})$ and $(Q_{i+1}, \mathcal{P}_{i+1}) := \mu_{v_i}(Q_i, \mathcal{P}_i)$ for $1 \leq i \leq r - 1$.

This assures that no loops or 2-cycles appear at each step, and that

$$(\mu_{v_i} \cdots \mu_{v_1}(T), \mu_{v_i} \cdots \mu_{v_1}(T)) \simeq \mathcal{J}(Q_{i+1}, \mathcal{P}_{i+1}) \text{ for } 1 \leq i \leq r - 1.$$

4. MUTATING AT LOOPS AND 2-CYCLES

In this section, we build on the theory of Section 3, in order to develop a method to mutate the quivers of cluster-tilting objects in algebraic 2-CY triangulated categories, having loops and 2-cycles. This is done under certain restrictions, as we now explain.

Let \mathcal{T} be an algebraic 2-CY triangulated category such that no cluster-tilting object in \mathcal{T} has loops and/or 2-cycles. Suppose that $\pi: \mathcal{T} \rightarrow \overline{\mathcal{T}}$ is a Galois covering of algebraic triangulated categories (then $\overline{\mathcal{T}}$ automatically also is 2-CY). Denote by G the group of k -linear automorphisms of \mathcal{T} such that $\overline{\mathcal{T}} = \mathcal{T}/G$. For an object $X \in \mathcal{T}$, we denote by $\overline{X} = \pi(X) \in \overline{\mathcal{T}}$. For an object $Y \in \overline{\mathcal{T}}$ we denote by $\overleftarrow{Y} = \pi^{-1}(Y) \in \mathcal{T}$.

We now show that we can lift cluster-tilting objects from $\overline{\mathcal{T}}$ to \mathcal{T} .

Proposition 4.1. *Let $\overleftarrow{\tau}\mathbb{T}$ be a cluster-tilting object in $\overline{\mathcal{T}}$. The orbit $\tau\mathbb{T} = \overleftarrow{\overleftarrow{\tau}\mathbb{T}}$ is a cluster-tilting object in \mathcal{T} .*

Proof. Using the bijections of the Hom-spaces between \mathcal{T} and $\overline{\mathcal{T}}$ given by π , we have that

$$0 = \text{Ext}_{\overleftarrow{\tau}}^1(\overleftarrow{\tau}\mathbb{T}, \overleftarrow{\tau}\mathbb{T}) \simeq \text{Ext}_{\tau}^1(\tau\mathbb{T}, \tau\mathbb{T})$$

and thus $\tau\mathbb{T}$ has no self extensions. (Here the first equality holds since $\overleftarrow{\tau}\mathbb{T}$ is just the sum of order of the covering many copies of $\overleftarrow{\tau}\mathbb{T}$.) Now assume that for X in \mathcal{T} , we have that $\text{Ext}_{\tau}^1(\tau\mathbb{T}, X) = 0$. This implies that $\text{Ext}_{\overleftarrow{\tau}}^1(\overleftarrow{\tau}\mathbb{T}, \overline{X}) = 0$, and thus \overline{X} is in $\text{add } \overleftarrow{\tau}\mathbb{T}$. But this just means that $\overleftarrow{\overline{X}}$ is in $\text{add } \tau\mathbb{T}$. In particular, X is in $\text{add } \tau\mathbb{T}$. \square

For the rest of the section, fix a basic cluster-tilting object $\overleftarrow{\tau}\mathbb{T} = \overleftarrow{\tau}\mathbb{T}_m \oplus \overleftarrow{\tau}\mathbb{T}_f$ in $\overline{\mathcal{T}}$, where the summand $\overleftarrow{\tau}\mathbb{T}_m$ is indecomposable. Define $\overline{\mathcal{D}} := \text{add } \overleftarrow{\tau}\mathbb{T}_f$ and $\overline{\mathcal{Z}} := {}^{\perp}\overline{\mathcal{D}}[1]$. Then the subfactor category $\overline{\mathcal{U}} := \overline{\mathcal{Z}}/[\overline{\mathcal{D}}]$ is a 2-CY triangulated category (see Subsection 2.7).

Write $\tau\mathcal{T} = \tau\mathcal{T}_m \oplus \tau\mathcal{T}_f$ for the lift of $\overline{\tau}\mathcal{T}$ in \mathcal{T} , such that $\tau\mathcal{T}_m = \overleftarrow{\tau}\overline{\mathcal{T}}_m$ and $\tau\mathcal{T}_f = \overleftarrow{\tau}\overline{\mathcal{T}}_f$. As in the previous paragraph, let $\mathcal{D} := \text{add } \tau\mathcal{T}_f$ and define \mathcal{Z} to be the subcategory ${}^\perp\mathcal{D}[1]$ of \mathcal{T} . Again, we have that the subfactor category $\mathcal{U} := \mathcal{Z}/[\mathcal{D}]$ forms a 2-CY triangulated category. Then we have the following.

Proposition 4.2. *The covering functor $\pi: \mathcal{T} \rightarrow \overline{\mathcal{T}}$ induces a triangle functor $\tilde{\pi}: \mathcal{U} \rightarrow \overline{\mathcal{U}}$ that is also a Galois covering.*

Proof. First, observe that π sends \mathcal{D} onto $\overline{\mathcal{D}}$ and \mathcal{Z} onto $\overline{\mathcal{Z}}$. Thus we have a well defined functor $\tilde{\pi}: \mathcal{U} \rightarrow \overline{\mathcal{U}}$. Second, note that π sends \mathcal{D} -approximations to $\overline{\mathcal{D}}$ -approximations, since $\tau\mathcal{T}_f = \overleftarrow{\tau}\overline{\mathcal{T}}_f$. Thus we have that $\tilde{\pi} \circ \langle 1 \rangle_{\mathcal{U}} = \langle 1 \rangle_{\overline{\mathcal{U}}} \circ \tilde{\pi}$, where $\langle 1 \rangle_{\mathcal{U}}$ and $\langle 1 \rangle_{\overline{\mathcal{U}}}$ denote the shift functors in the categories \mathcal{U} and $\overline{\mathcal{U}}$, respectively.

Now to see that $\tilde{\pi}$ is a triangle functor, let $X \rightarrow Y \rightarrow Z \rightarrow X\langle 1 \rangle$ be a standard triangle in \mathcal{U} . This triangle comes from the following commutative diagram of triangles in \mathcal{T}

$$\begin{array}{ccccccc} X & \longrightarrow & Y & \longrightarrow & Z & \longrightarrow & X[1] \\ \downarrow 1_X & & \downarrow & & \downarrow & & \downarrow 1_{X[1]} \\ X & \xrightarrow{\alpha_X} & D_X & \xrightarrow{\beta_X} & X\langle 1 \rangle & \longrightarrow & X[1] \end{array}$$

where the morphism α_X (resp. β_X) is a left (resp. right) \mathcal{D} -approximation (see Subsection 2.7). This diagram descends via π to the following commutative diagram of triangles in $\overline{\mathcal{U}}$

$$\begin{array}{ccccccc} \overline{X} & \longrightarrow & \overline{Y} & \longrightarrow & \overline{Z} & \longrightarrow & \overline{X}[1] \\ \downarrow 1_{\overline{X}} & & \downarrow & & \downarrow & & \downarrow \\ \overline{X} & \xrightarrow{\pi(\alpha_X)} & \overline{D}_X & \xrightarrow{\pi(\beta_X)} & \overline{X}\langle 1 \rangle & \longrightarrow & \overline{X}[1] \end{array}$$

where the morphism $\pi(\alpha_X)$ (resp. $\pi(\beta_X)$) is a left (resp. right) $\overline{\mathcal{D}}$ -approximation. Hence we obtain a standard triangle $\overline{X} \rightarrow \overline{Y} \rightarrow \overline{Z} \rightarrow \overline{X}\langle 1 \rangle$ in $\overline{\mathcal{U}}$. It is not difficult to see that G acts freely and transitively on the fibers of $\tilde{\pi}$. Thus $\tilde{\pi}$ is a Galois covering. \square

We now define the setup under which we can mutate at loops and 2-cycles.

Setup 4.3 (for mutation at loops and 2-cycles). Let $\pi: \mathcal{T} \rightarrow \overline{\mathcal{T}}$ be a Galois covering of algebraic 2-CY triangulated categories. Assume we have a (basic) cluster-tilting object $\overline{\tau}\mathcal{T} = \overline{\tau}\mathcal{T}_m \oplus \overline{\tau}\mathcal{T}_f$ in $\overline{\mathcal{T}}$, where the summand $\overline{\tau}\mathcal{T}_m$ is indecomposable. We write $\tau\mathcal{T}_m = \overleftarrow{\tau}\overline{\mathcal{T}}_m$, $\tau\mathcal{T}_f = \overleftarrow{\tau}\overline{\mathcal{T}}_f$, and $\tau\mathcal{T} = \overleftarrow{\tau}\overline{\mathcal{T}}$ ($= \tau\mathcal{T}_m \oplus \tau\mathcal{T}_f$). If we denote by Q and \overline{Q} the quivers of $(\tau\mathcal{T}, \tau\mathcal{T})$ and $(\overline{\tau}\mathcal{T}, \overline{\tau}\mathcal{T})$, respectively, then the covering functor $\pi: \mathcal{T} \rightarrow \overline{\mathcal{T}}$ induces a covering of quivers, which we also denote by $\pi: Q \rightarrow \overline{Q}$. Note that $\overline{Q}_{m,m}$, the quiver of $(\overline{\tau}\mathcal{T}_m, \overline{\tau}\mathcal{T}_m)/[\overline{\tau}\mathcal{T}_f]$, now is a single vertex, possibly with a loop and/or 2-cycles adjacent to this vertex in \overline{Q} .

We assume the category \mathcal{T} and the cluster-tilting objects $\tau\mathcal{T} = \tau\mathcal{T}_m \oplus \tau\mathcal{T}_f$ and $\tau\mathcal{T}' = \tau\mathcal{T}_m\langle 1 \rangle_{\mathcal{U}} \oplus \tau\mathcal{T}_f$ in \mathcal{T} , are as in Setup 3.4.

Remark 4.4. Observe that under the setup above, the quiver $Q_{m,m}$ is a (possibly disjoint union of) cycle(s) of length $l \geq 3$ in Q with minimal relations given by the paths of length l .

Remark 4.5. In the setup above, note that the fact that the endomorphism ring of τT is given by a quiver with potential leads to the endomorphism ring of $\overline{\tau} T$ also being given by a quiver with potential, provided the characteristic of k does not divide order of the covering.

If the characteristic of k divides the order of the covering it can happen that some cyclic derivatives for the candidate potential on \overline{Q} vanish, even though there should be relations.

Consider for instance the quiver with just one vertex, and a loop α attached to it, subject to the relation $\alpha^n = 0$. If the characteristic of k does not divide $n + 1$ then this relation is given by the potential α^{n+1} , but if the characteristic of k divides $n + 1$ then this potential does not give rise to any relations.

We now present the main theorem of this section.

Theorem 4.6 (Mutation at loops and 2-cycles). *Let $\pi: \mathcal{T} \rightarrow \overline{\mathcal{T}}$ be a Galois covering of algebraic 2-CY triangulated categories. Let $\overline{\tau} T = \overline{\tau} T_m \oplus \overline{\tau} T_f$ be a (basic) cluster-tilting object in $\overline{\mathcal{T}}$ with $\overline{\tau} T_m$ indecomposable and denote by $\overline{\tau} T'_m$ the other complement of the almost complete cluster-tilting object $\overline{\tau} T_f$. Write $\tau T = \tau T_m \oplus \tau T_f$ for the lift of $\overline{\tau} T$ to \mathcal{T} via π . Suppose that the conditions of Setup 4.3 are satisfied. Then, using the same notation as in Setup 4.3, we have that $\overline{\tau} T' = \overline{\tau} T'_m \oplus \overline{\tau} T_f = \pi(\tau T')$. Furthermore, if \overline{Q} denotes the quiver of $(\overline{\tau} T', \overline{\tau} T')$, then $\overline{Q}' = \pi(Q')$.*

Proof. From the proof of Proposition 4.2, we see that mutating in $\overline{\mathcal{T}}$ corresponds to mutating in \mathcal{T} as in Construction 3.1.

If the quiver of $\overline{\tau} T$ has loops or 2-cycles, these disappear when lifting to τT since by assumption cluster-tilting objects in \mathcal{T} have no loops or 2-cycles. This allows us to mutate via the cover as follows: We mutate Q at $Q_{m,m}$ using Theorem 3.5 to obtain Q' . Then \overline{Q}' , the quiver of $\overline{\tau} T'$, is given by $\pi(Q')$. \square

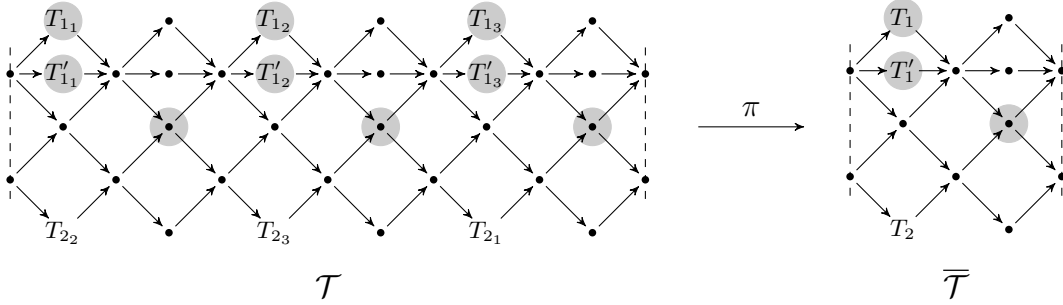
Remark 4.7. Consider the case when $\overline{Q}_{m,m}$ does not have a loop, but there are possibly 2-cycles adjacent to the only vertex in $\overline{Q}_{m,m}$. Then it is not difficult to see that the subfactor category \mathcal{U} of \mathcal{T} is just the product of cluster categories of type A_1 . This means that the quiver $Q_{m,m}$ is a disjoint union of isolated vertices, and FZ-quiver mutation at all the vertices in $Q_{m,m}$ (in any order) in the cover gives the correct answer. Then project back using π .

Remark 4.8. When \mathcal{T} is a cluster category, we know that the relations of any cluster-tilted algebra are determined by its quiver ([BIRSm, 5.11]). In this case, the relations of the 2-CY tilted algebra $(\overline{\tau} T, \overline{\tau} T)$ are uniquely determined by the cover.

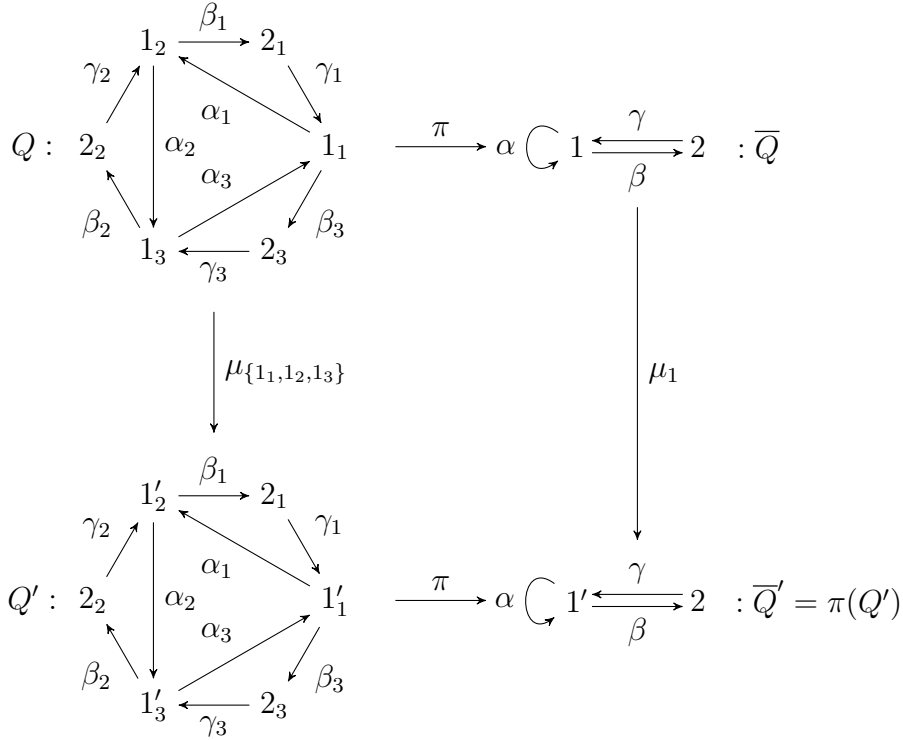
We now present an example that illustrates the procedure above.

Example 4.9. Let $\mathcal{T} = \mathcal{D}_{6,1}$ be the cluster category of \mathbb{D}_6 and $\overline{\mathcal{T}} = \mathcal{D}_{6,3}$ the covering of order 3 (see the diagram below – also see Definition 5.3 for notation). It is not hard to see that we have a Galois covering $\pi: \mathcal{T} \rightarrow \overline{\mathcal{T}}$. Let $\overline{\tau} T = T_1 \oplus T_2$ be the cluster-tilting object in $\overline{\mathcal{T}}$ and denote its lift to \mathcal{T} by $\tau T = \bigoplus_{i,j} T_{i,j}$ for $1 \leq i \leq 2$ and $1 \leq j \leq 3$, both shown in the figure below. Similarly, let $\overline{\tau} T' = T'_1 \oplus T_2$ be the

cluster-tilting object obtained by replacing T_1 in $\overline{\tau}T$. We can also obtain $\overline{\tau}T'$ by replacing $\bigoplus_{j=1}^3 T_{1_j}$ by $\bigoplus_{j=1}^3 T'_{1_j}$ in τT and then projecting to $\overline{\mathcal{T}}$ via π , as illustrated in the figure below.



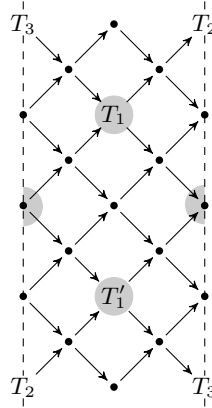
In the figure above, the subfactor categories \mathcal{U} and $\overline{\mathcal{U}}$ are presented in light grey. They correspond to the 2-CY triangulated categories $\mathcal{A}_{3,1}$ and $\mathcal{A}_{3,3}$, respectively (see Definition 5.3). At the level of quivers, we obtain the following picture:



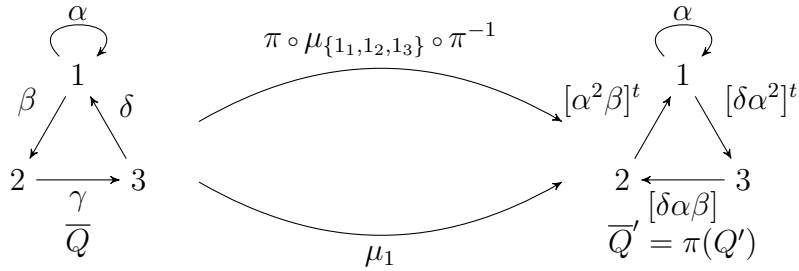
Here, Q and \overline{Q} denote the quivers of the endomorphism rings of τT and $\overline{\tau}T$ respectively. Applying the mutation rule for cycles to Q we obtain the quiver Q' . Then mutating \overline{Q} at vertex 1 we obtain \overline{Q}' , which is isomorphic to the quiver given by $\pi(Q')$. The relations on \overline{Q} (resp. \overline{Q}') are determined by the potential of Q (resp. Q').

Example 4.10. Let $\mathcal{T} = \mathcal{A}_{9,1}$ be the cluster category of \mathbb{A}_9 and $\overline{\mathcal{T}} = \mathcal{A}_{9,3}$ the covering of order 3 (see the diagram below – also see Definition 5.3 for notation). Then we have a Galois covering $\pi: \mathcal{T} \rightarrow \overline{\mathcal{T}}$ of 2-CY triangulated categories. Let $\overline{\tau}T = T_1 \oplus T_2 \oplus T_3$ be the cluster-tilting object in $\overline{\mathcal{T}}$ shown below. We denote by $\overline{\tau}T' = T'_1 \oplus T_2 \oplus T_3$ the resulting cluster-tilting object obtained by replacing T_1 in

$\overline{\tau}T$.



The lift τT (resp. $\tau T'$) of $\overline{\tau}T$ (resp. $\overline{\tau}T'$) to \mathcal{T} is shown in Example 3.3. The mutation of Q , the quiver of τT , at the cycle $\{1_1, 1_2, 1_3\}$ in order to obtain Q' , the quiver of $\tau T'$, is illustrated in Example 3.6. Then after mutating \overline{Q} at vertex 1 we obtain \overline{Q}' , which is isomorphic to the quiver given by $\pi(Q')$. Therefore we have the following commutative diagram.



The relations on \overline{Q} (resp. \overline{Q}') are determined by the potential of Q (resp. Q').

5. 2 CY-TILTED ALGEBRAS OF FINITE TYPE

In this section, we give a description of the mutation classes of the 2-CY tilted algebras (which are not cluster-tilted) coming from standard algebraic 2-CY triangulated categories with a finite number of indecomposables. (A category is called standard if it is equivalent to the mesh category of its Auslander-Reiten quiver.) We will see that these types of algebras always satisfy our setup for mutating at loops. Thus by using our mutation rule developed in the previous section, we will be able to mutate at any vertex.

Let k be an algebraically closed field. The 2-CY tilted algebras of finite type appear as endomorphism rings of cluster-tilting objects in k -linear 2-CY triangulated categories \mathcal{T} with a finite number of indecomposables. In [BIKR] the authors prove that the existence of cluster-tilting objects in these categories follows from the shape of their AR-quiver. These shapes were described in [A, XZ].

First, let us fix a numbering and an orientation of the simply-laced Dynkin quivers.

Definition 5.3. Let ℓ and n be two positive integers such that ℓ divides n . We denote by $\mathcal{A}_{n,\ell}$ (resp. $\mathcal{D}_{n,\ell}$ and $\mathcal{E}_{n,\ell}$) the standard algebraic 2-CY triangulated category having AR-quiver $\mathbb{Z}\mathbb{A}_n/g$ (resp. $\mathbb{Z}\mathbb{D}_n/g$ and $\mathbb{Z}\mathbb{E}_n/g$), where in each case we have that $g^\ell = F$.

Remark 5.4. This definition only makes sense if we ask our triangulated category to be standard and algebraic. By [A, 7.0.5], it is known that these categories are unique up to a triangle isomorphism. With the above definitions, the cluster categories of type \mathbb{A}_n , \mathbb{D}_n and \mathbb{E}_8 are denoted by $\mathcal{A}_{n,1}$, $\mathcal{D}_{n,1}$ and $\mathcal{E}_{8,1}$ respectively.

We can now give a simpler reformulation of Theorem 5.2 for the standard algebraic case.

Theorem 5.5. *Let \mathcal{T} be a standard algebraic 2-CY triangulated category, not a cluster category, with a finite number of indecomposables. Then \mathcal{T} has a cluster-tilting object if and only if \mathcal{T} is either the category $\mathcal{A}_{3n,3}$ for some $n \geq 1$, the category $\mathcal{D}_{n\ell,\ell}$ for n and ℓ such that $n\ell \geq 4$, or the category $\mathcal{E}_{8,2}$.*

Proof. This is a direct consequence of [A, 7.0.5] and Theorem 5.2. \square

The following definition is useful for the description of the 2-CY tilted algebras of finite type. It was first introduced in [V] in order to describe the cluster-tilted algebras of type \mathbb{D} .

Definition 5.6. Let Q be the quiver of a cluster-tilted algebra of type \mathbb{A} . A vertex of Q is called a *connecting vertex* if

- (a) there are at most two arrows adjacent to it, and
- (b) whenever there are two arrows adjacent to it, the vertex is on a 3-cycle.

We are now ready to present the main theorem of this section.

Theorem 5.7. *Let T be a cluster-tilting object in a standard algebraic 2-CY triangulated category \mathcal{T} of finite type, not a cluster category. Then $\text{End}_{\mathcal{T}}(T)$ is depicted in Figure 5.1.*

Proof. Observe that we have a covering functor $\pi: \mathcal{C}_\Delta \rightarrow \mathcal{T}$, where \mathcal{C}_Δ is a cluster category of Dynkin type Δ . We proceed by finding the cluster-tilted algebras in \mathcal{C}_Δ which are a cover of the 2-CY tilted algebras in \mathcal{T} . By Theorem 5.5 we have three cases:

Case $\mathcal{T} = \mathcal{A}_{3n,3}$. Using the geometric description of the cluster category of type \mathbb{A} , we know that cluster-tilting objects correspond to triangulations of a regular $(3n+3)$ -gon (see [CCS, I]). Observe that the automorphism g corresponds to a rotation by $2\pi/3$. We want to find all the triangulations of the polygon invariant under g .

Assume we are given such a g -invariant triangulation. Let d be the longest diagonal which is part of the triangulation. If it covers an angle of more than $2\pi/3$ then the diagonals d and $g \cdot d$ intersect in their interior, a contradiction. If all diagonals cover an angle of less than $2\pi/3$ then the shape which contains the center of the polygon cannot be a triangle, also a contradiction. Thus d covers an angle of exactly $2\pi/3$, and d , $g \cdot d$, and $g^2 \cdot d$ form a regular triangle in the center of the regular $(3n+3)$ -gon.

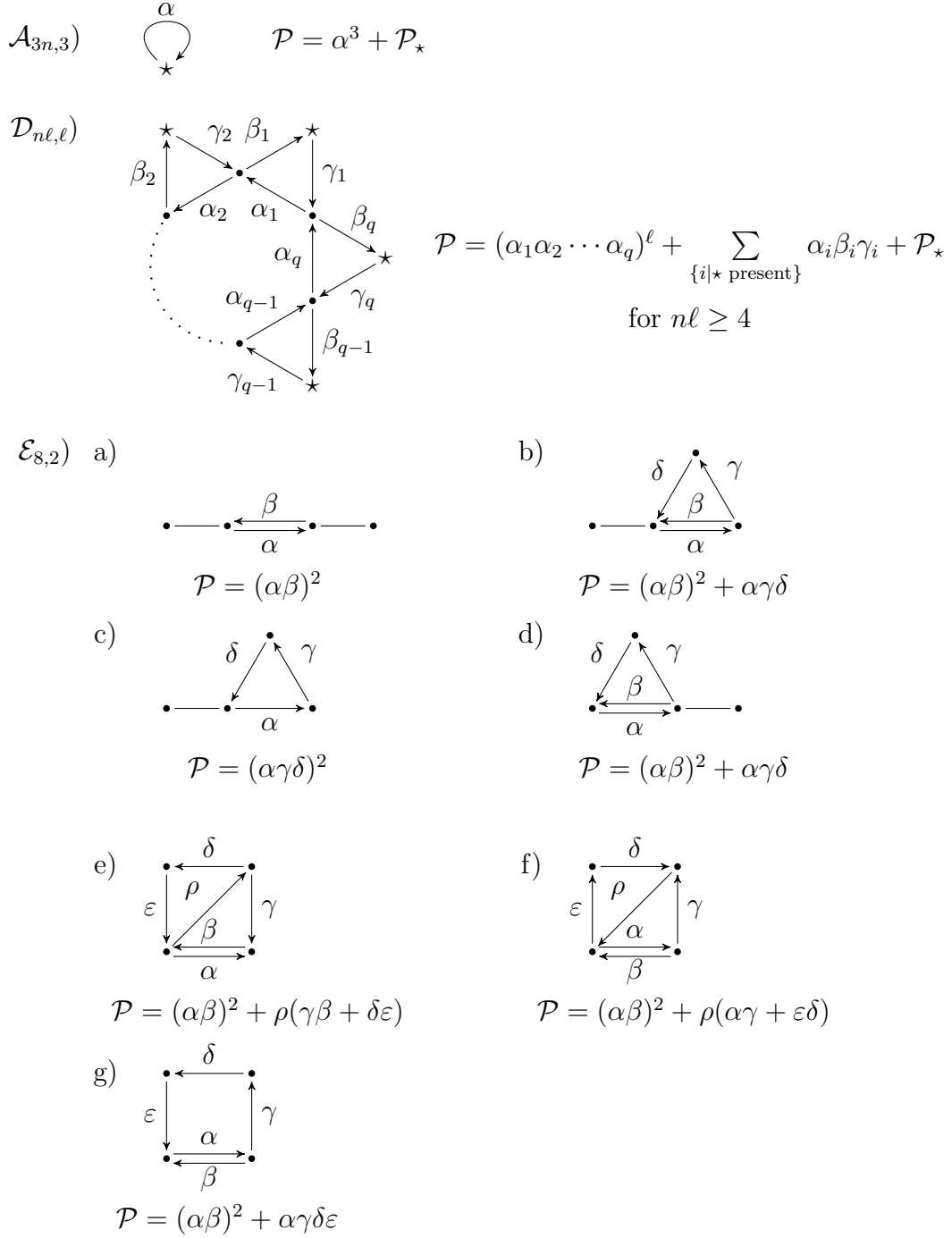


FIGURE 5.1. These are the 2-CY tilted algebras of finite type that are not cluster-tilted, organized by their mutation class. The relations are given by the potentials \mathcal{P} , unless (see Remark 4.5) the characteristic is 3 in case $\mathcal{A}_{3n,3}$, 2 in case $\mathcal{E}_{8,2}$, or divides ℓ in case $\mathcal{D}_{nl,\ell}$ – in these cases the relations are similar but do not come from the potential. For the cases $\mathcal{A}_{3n,3}$ and $\mathcal{D}_{nl,\ell}$, the vertex \star is a connecting vertex (see Definition 5.6) where a cluster-tilted algebra of type \mathbb{A} is glued, and the term \mathcal{P}_\star corresponds to the sum of the potentials of all cluster-tilted algebras of type \mathbb{A} attached at \star . In case $\mathcal{D}_{nl,\ell}$, the vertices \star may or may not be present, and thus, the corresponding β and γ arrows disappear.

Next, we note that the remaining diagonals correspond to three identical triangulations of an $(n + 2)$ -gon, that is, a cluster-tilting object in the cluster category of type \mathbb{A}_{n-1} .

Now projecting back to \mathcal{T} , we obtain a 2-CY tilted algebra with a loop α corresponding to the orbit of the diagonal d , attached to a cluster-tilted algebra of type \mathbb{A} . Since the orbit of the diagonal d is a triangle, the loop satisfies the relation $\alpha^2 = 0$, thus obtaining the quiver with relations depicted in Figure 5.1 $\mathcal{A}_{3n,n}$.

Case $\mathcal{T} = \mathcal{D}_{nl,\ell}$. We can assume $nl \geq 4$. In this case, cluster-tilting objects in the cluster category of \mathbb{D}_{nl} correspond to (tagged) triangulations of a punctured nl -gon (see [S]). Here the automorphism g corresponds to a rotation by $2\pi/\ell$ composed with ϕ^n where ϕ is the automorphism that exchanges the tagged diagonals with non-tagged diagonals.

Assume we are given a g -invariant triangulation of the punctured nl -gon. By definition there is at least one diagonal connecting the puncture to the polygon. It follows that there are at least ℓ vertices of the polygon connected to the puncture by diagonals (the ℓ g -translates of any given one).

It follows that no vertex is connected to the puncture by more than one diagonal (hence we may ignore the question if edges are tagged or not).

Now consider all diagonals connecting vertices of the polygon to the puncture. Clearly they form a cycle of length $q\ell$ in the quiver of the cluster tilted algebra of type \mathbb{D}_{nl} , and hence a cycle of length q in the quiver of our 2-CY tilted algebra of type $\mathcal{D}_{nl,\ell}$.

If two consecutive such diagonals start in consecutive vertices of the polygon, then the corresponding arrow of the q -cycle is not involved in any further cycles of the quiver of the 2-CY tilted algebra.

If two consecutive diagonals ending in the puncture start in vertices of the polygon which are further apart, then these vertices are connected by another diagonal. Moreover there is some triangulation of the part of the polygon cut off by this other diagonal. In the quiver of the 2-CY tilted algebra this means that the arrow of the q -cycle is involved in one further triangle, which connects it to the connecting vertex of some quiver of a cluster tilted algebra of type \mathbb{A} .

Case $\mathcal{T} = \mathcal{E}_{8,2}$. This is a finite combinatorial task. It is simplified by the following observations:

- (a) Numbering the τ -orbits starting from the top most orbit and below, we have 8 orbits, say $\sigma_1, \dots, \sigma_8$ (see Figure 5.2).
- (b) The orbits σ_1, σ_2 and σ_8 are the only ones having exceptional objects. To a cluster-tilting object we can associate a triple (a_1, a_2, a_8) of non-negative integers, where a_i denotes the number of indecomposable summands in the orbit σ_i .
- (c) $a_i \in \{0, 1, 2\}$ for $i \in \{1, 2, 8\}$.

We consider the numbering of the indecomposable objects of $\mathcal{E}_{8,2}$ as in Figure 5.2. The possible cluster-tilting objects up to a τ -shift are illustrated in Table 5.1. Their endomorphism rings are depicted in Figure 5.1 $\mathcal{E}_{8,2}$.

Thus the assertion from the theorem follows. \square

Remark 5.8. Using Theorem 4.6, we can now mutate at any vertex for a 2-CY tilted algebra of finite type (see for instance Examples 4.9 and 4.10). One can check that

in these finite 2-CY categories, all cluster-tilting objects are mutation connected. We illustrate the mutation graph of $\mathcal{E}_{8,2}$ in Figure 5.3.

T	$\text{End}_{\mathcal{T}}(T)$	Type
$(0 \oplus 24, -, 3 \oplus 27)$	g	$(2,0,2)$
$(24 \oplus 48, -, 27 \oplus 51)$	g	$(2,0,2)$
$(0 \oplus 24, -, 3 \oplus 43)$	f	$(2,0,2)$
$(24 \oplus 48, -, 3 \oplus 27)$	f	$(2,0,2)$
$(0 \oplus 24, -, 27 \oplus 51)$	e	$(2,0,2)$
$(24 \oplus 48, -, 11 \oplus 51)$	e	$(2,0,2)$
$(24, 28, 11 \oplus 51)$	c	$(1,1,2)$
$(32, 28, 11 \oplus 51)$	c	$(1,1,2)$
$(0 \oplus 24, 28, 51)$	d	$(2,1,1)$
$(0 \oplus 32, 28, 51)$	d	$(2,1,1)$
$(24 \oplus 56, 28, 11)$	b	$(2,1,1)$
$(32 \oplus 56, 28, 11)$	b	$(2,1,1)$
$(0 \oplus 24, 28 \oplus 60, -)$	a	$(2,2,0)$
$(0 \oplus 32, 28 \oplus 60, -)$	a	$(2,2,0)$
$(24 \oplus 56, 28 \oplus 60, -)$	a	$(2,2,0)$
$(32 \oplus 56, 28 \oplus 60, -)$	a	$(2,2,0)$

TABLE 5.1. Possible cluster-tilting objects up to τ -shift in $\mathcal{E}_{8,2}$.

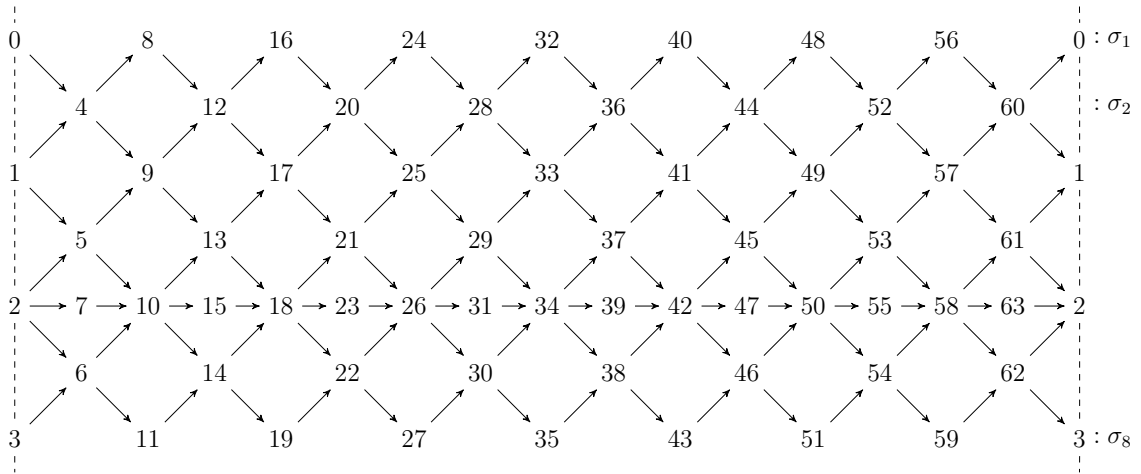


FIGURE 5.2. Numbering of the vertices in the AR-quiver of the 2-CY triangulated category $\mathcal{E}_{8,2}$.

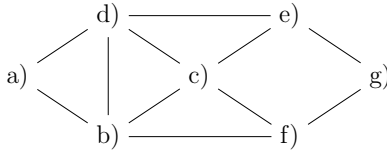


FIGURE 5.3. Mutation component of the 2-CY triangulated category $\mathcal{E}_{8,2}$.

Remark 5.9. If the characteristic of k allows to see our 2-CY tilted algebra to be given by a quiver with potential (that is it does not divide the order of the covering – see Remark 4.5), and the quiver does not contain any loops, then we are in a setup where mutation of quivers with potential is defined.

Under these assumptions mutation of quivers with potential and our mutation rule via coverings yield the same result. For instance this applies to the 2-CY tilted algebras of type $\mathcal{E}_{8,2}$, provided the characteristic of k is odd or 0.

6. APPENDIX

Proof of Theorem 3.5. We have now the following cases:

- $M'_{m,m}$. Using that $A^{l-1}A = AA^t = 1$, we see that $-A^{-1}(A - A^t)(-A) = A - A^t$. Thus, the arrows in $Q_{m,m}$ remain unchanged.
- $M'_{m,f}$. Observing that $M'_{m,f} = -(M'_{f,m})^t$, it suffices to calculate just one of them.

$$\begin{aligned}
M'_{f,m} &= \left(\left(\sum_{i=0}^{l-2} C_i \sum_{j=0}^i A^j \right) (A - A^t) + C - B^t \right) (-A) \\
&= \sum_{i=0}^{l-2} C_i \underbrace{\left(- \sum_{j=0}^i A^{j+2} + \sum_{j=0}^i A^j - A \right)}_{=1 - A^{i+1} - A^{i+2}} + \sum_{i=0}^{l-3} \underbrace{B_i^t A}_{=C_i A^{i+2}} + B_{l-2}^t A \\
&= - \sum_{i=0}^{l-2} C_i A^{i+1} + \sum_{i=0}^{l-2} C_i - C_{l-2} \underbrace{A^l}_{=1} + B_{l-2}^t A \\
&= - \sum_{i=0}^{l-3} \underbrace{C_i A^{i+1}}_{B_i^t} - C_{l-2} A^{l-1} + \sum_{i=0}^{l-3} C_i + B_{l-2}^t A \\
&= \sum_{i=0}^{l-3} C_i - \sum_{i=0}^{l-3} B_i^t + (A^{l-1} B_{l-2})^t - C_{l-2} A^{l-1}.
\end{aligned}$$

Thus any arrow γ in C_i , with $0 \leq i \leq l-3$ remains unchanged. Let γ be an arrow in C_{l-2} . There is a path $\gamma\alpha^{l-1}$. We replace this arrow γ by an arrow from the end of $\gamma\alpha^{l-1}$ to the start of γ . Apply the dual process for arrows in $Q_{m,f}$.

- $M'_{f,f}$. This part of the matrix is composed of the following summands.

$$\begin{aligned}
M'_{f,f} &= \left(\sum_{i=0}^{l-2} C_i \sum_{j=0}^i A^j \right) (A - A^{-1}) \left(\sum_{i=0}^{l-2} C_i \sum_{j=0}^i A^j \right)^t \\
&\quad + \left(\sum_{i=0}^{l-2} C_i \sum_{j=0}^i A^j \right) (B - C^t) + (C - B^t) \left(\sum_{i=0}^{l-2} C_i \sum_{j=0}^i A^j \right)^t + (D - D^t).
\end{aligned}$$

We divide the calculations in 4 steps:

1) Summands of the type $C_{l-2}(\cdots)C_{l-2}^t$. Denote by $\Sigma = \sum_{j=0}^{l-1} A^j$.

$$\begin{aligned} & C_{l-2} \left(\sum_{j=0}^{l-2} A^j \right) (A - A^{-1}) \left(C_{l-2} \sum_{j=0}^{l-2} A^j \right)^t + C_{l-2} \left(\sum_{j=0}^{l-2} A^j \right) (-C_{l-2}^t) \\ & + C_{l-2} \left(C_{l-2} \sum_{j=0}^{l-2} A^j \right)^t \\ & = C_{l-2} ((\Sigma - A^{-1})(A - A^{-1})(\Sigma - A) - (\Sigma - A^{-1}) + (\Sigma - A)) C_{l-2}^t \\ & = C_{l-2} (A - A^{-1} + A^{-1} - A) C_{l-2}^t = 0. \end{aligned}$$

2) Summands including C_{l-2} and B_{l-2} .

$$C_{l-2} \left(\sum_{j=0}^{l-2} A^j \right) B_{l-2} - \left(C_{l-2} \left(\sum_{j=0}^{l-2} A^j \right) B_{l-2} \right)^t.$$

3) Summands including C_{l-2} or B_{l-2} and terms of lower indices.

$$\begin{aligned} & C_{l-2} \left(\sum_{j=0}^{l-2} A^j \right) (A - A^{-1}) \left(\sum_{i=0}^{l-3} C_i \sum_{j=0}^i A^j \right)^t \\ & + \left(\sum_{i=0}^{l-3} C_i \sum_{j=0}^i A^j \right) (A - A^{-1}) \left(\sum_{j=0}^{l-2} A^{-j} \right) C_{l-2}^t \\ & + C_{l-2} \left(\sum_{j=0}^{l-2} A^j \right) \left(\sum_{i=0}^{l-3} B_i - \sum_{i=0}^{l-3} C_i^t \right) + \left(\sum_{i=0}^{l-3} \sum_{j=0}^i C_i A^j \right) (B_{l-2} - C_{l-2}^t) \\ & + (C_{l-2} - B_{l-2}^t) \left(\sum_{i=0}^{l-3} \sum_{j=0}^i A^{-j} C_i^t \right) + \left(\sum_{i=0}^{l-3} C_i - \sum_{i=0}^{l-3} B_i^t \right) \left(\sum_{j=0}^{l-2} A^{-j} \right) C_{l-2}^t \\ & = \sum_{i=0}^{l-3} C_{l-2} \left[(\Sigma - A^{-1})(A - A^{-1}) \left(\sum_{j=0}^i A^{-j} \right) + (\Sigma - A^{-1})(A^{-i-1} - 1) + \left(\sum_{j=0}^i A^{-j} \right) \right] C_i^t \\ & + \sum_{i=0}^{l-3} C_i \left[\left(\sum_{j=0}^i A^j \right) (A - A^{-1})(\Sigma - A) + \left(\sum_{j=0}^i A^j \right) (-1) + (1 - A^{i+1})(\Sigma - A) \right] C_{l-2}^t \\ & - \sum_{i=0}^{l-3} B_{l-2}^t \left(\sum_{j=0}^i A^{-j} \right) C_i^t + \sum_{i=0}^{l-3} C_i \left(\sum_{j=0}^i A^j \right) B_{l-2} \\ & = \sum_{i=0}^{l-3} \left[C_{l-2} \left(-A^{-1}(A + 1 - A^{-i} - A^{-i-1}) - A^{-i-2} + A^{-1} + \sum_{j=0}^i A^{-j} \right) C_i^t \right. \\ & \left. + C_i \left((A^{i+1} + A^i - 1 - A^{-1})(-A) - \sum_{j=0}^i A^j - A + A^{i+2} \right) C_{l-2}^t \right] \end{aligned}$$

$$\begin{aligned}
& + \sum_{i=0}^{l-3} \left[C_i \left(\sum_{j=0}^i A^j \right) B_{l-2} - B_{l-2}^t \left(\sum_{j=0}^i A^{-j} \right) C_i^t \right] \\
& = \sum_{i=0}^{l-3} \left[C_{l-2} \left(\sum_{j=1}^{i+1} A^{-j} \right) C_i^t + C_i \left(- \sum_{j=1}^{i+1} A^j \right) C_{l-2}^t \right] \\
& \quad + \sum_{i=0}^{l-3} \left[C_i \left(\sum_{j=0}^i A^j \right) B_{l-2} - \left(C_i \left(\sum_{j=0}^i A^j \right) B_{l-2} \right)^t \right] \\
& = \sum_{i=0}^{l-3} \left[C_{l-2} \left(\sum_{j=0}^i A^j \right) B_i - \left(C_{l-2} \left(\sum_{j=0}^i A^j \right) B_i \right)^t \right. \\
& \quad \left. + C_i \left(\sum_{j=0}^i A^j \right) B_{l-2} - \left(C_i \left(\sum_{j=0}^i A^j \right) B_{l-2} \right)^t \right]
\end{aligned}$$

4) Summands without terms of index $l-2$.

$$\begin{aligned}
& \sum_{i_1=0}^{l-3} \sum_{i_2=0}^{l-3} C_{i_1} \left[\left(\sum_{j_1=0}^{i_1} A^{j_1} \right) (A - A^{-1}) \left(\sum_{j_2=0}^{i_2} A^{-j_2} \right) \right. \\
& \quad \left. + \left(\sum_{j_1=0}^{i_1} A^{j_1} \right) (A^{-i_2-1} - 1) + (1 - A^{i_1+1}) \left(\sum_{j_2=0}^{i_2} A^{-j_2} \right) \right] C_{i_2}^t \\
& = \sum_{i_1=0}^{l-3} \sum_{i_2=0}^{l-3} C_{i_1} \left[\sum_{j_1=0}^{i_1-1} A^{j_1+1} + \sum_{j_2=0}^{i_2} A^{i_1+1-j_2} - \sum_{j_1=0}^{i_1} A^{j_1-1-i_2} \right. \\
& \quad \left. - \sum_{j_2=0}^{i_2-1} A^{-1-j_2} + \sum_{j_1=0}^{i_1} A^{j_1-1-i_2} - \sum_{j_1=0}^{i_1} A^{j_1} + \sum_{j_2=0}^{i_2} A^{-j_2} - \sum_{j_2=0}^{i_2} A^{i_1+1-j_2} \right] C_{i_2}^t \\
& = \sum_{i_1=0}^{l-3} \sum_{i_2=0}^{l-3} C_{i_1} [-1 + 1] C_{i_2}^t = 0.
\end{aligned}$$

Thus, to sum up, we have

$$\begin{aligned}
M'_{f \rightarrow f} & = D - D^t + C_{l-2} \left(\sum_{j=0}^{l-2} A^j \right) B_{l-2} - \left(C_{l-2} \left(\sum_{j=0}^{l-2} A^j \right) B_{l-2} \right)^t \\
& \quad + \sum_{i=0}^{l-3} \left[C_{l-2} \left(\sum_{j=0}^i A^j \right) B_i - \left(C_{l-2} \left(\sum_{j=0}^i A^j \right) B_i \right)^t \right. \\
& \quad \left. + C_i \left(\sum_{j=0}^i A^j \right) B_{l-2} - \left(C_i \left(\sum_{j=0}^i A^j \right) B_{l-2} \right)^t \right].
\end{aligned}$$

Hence, we keep all arrows in $Q_{f,f}$, and add an arrow for each composition $\gamma\alpha^i\beta$ where $0 \leq i \leq l-2$ and $\gamma \in Q_{f,m}$, $\alpha \in Q_{m,m}$, $\beta \in Q_{m,f}$ such that

- neither $\gamma\alpha^i$ nor $\alpha^i\beta$ is zero or factors through an arrow in $Q_{f,f}$,

- either $\gamma \in C_{l-2}$ or $\beta \in B_{l-2}$, i.e. either γ or β has no extra relations with the minimal cycle of $Q_{m,m}$.
- Finally, remove all loops and 2-cycles from the mutated quiver.

REFERENCES

- [A] Claire Amiot. On the structure of triangulated categories with finitely many indecomposables. *Bull. Soc. Math. France* 135(2007), no. 3, 435–474.
- [ARS] M. Auslander, I. Reiten, S. Smalø. Representation theory of artin algebras. Cambridge University Press 1995.
- [BG] K. Bongartz and P. Gabriel, Covering spaces in representation-theory *Invent. Math.* 65 (1981/1982), no. 3, 331–378.
- [BIKR] Igor Burban, Osamu Iyama, Bernhard Keller and Idun Reiten. Cluster tilting for one-dimensional hypersurface singularities. *Adv. Math.*, 217(6):2443–2484, 2008.
- [BIRSc] Aslak B. Buan, Osamu Iyama, Idun Reiten, and Jeanne Scott. Cluster structures for 2-Calabi-Yau categories and unipotent groups. *Compos. Math.* 145 (2009), no. 4, 1035–1079.
- [BIRSm] Aslak B. Buan, Osamu Iyama, Idun Reiten, and David Smith. Mutation of cluster-tilting objects and potentials. Preprint, to appear in *American J. Math.*
- [BMR1] Aslak Bakke Buan, Robert J. Marsh, and Idun Reiten. Cluster-tilted algebras of finite representation type. *J. Algebra*, 306(2):412–431, 2006.
- [BMR2] Aslak Bakke Buan, Robert J. Marsh, and Idun Reiten. Cluster-tilted algebras. *Trans. Amer. Math. Soc.*, 359(1):323–332 (electronic), 2007.
- [BMR3] Aslak Bakke Buan, Robert J. Marsh, and Idun Reiten. Cluster mutation via quiver representations. *Comment. Math. Helv.*, 83(1):143–177, 2008.
- [BMRRT] Aslak Bakke Buan, Robert Marsh, Markus Reineke, Idun Reiten, and Gordana Todorov. Tilting theory and cluster combinatorics. *Adv. Math.*, 204(2):572–618, 2006.
- [BOW1] Marco A. Bertani-Økland, Steffen Opperman and Anette Wrålsen. Finding a cluster-tilting object for a representation finite cluster-tilted algebra. *Colloq. Math.*, 121(2): 249–263, 2010.
- [BOW2] Marco Angel Bertani-Økland, Steffen Oppermann, and Anette Wrålsen. Graded mutation in cluster categories coming from hereditary categories with a tilting object. Preprint, arXiv:1009.4812v1.
- [CCS] P. Caldero, F. Chapoton, and R. Schiffler. Quivers with relations arising from clusters (A_n case). *Trans. Amer. Math. Soc.* 358 (2006), no. 3, 1347–1364 (electronic).
- [DWZ] Harm Derksen, Jerzy Weyman, and Andrei Zelevinsky Quivers with potentials and their representations. I. Mutations *Selecta Math. (N.S.)* 14 (2008), no. 1, 59–119.
- [FZ] Sergey Fomin and Andrei Zelevinsky. Cluster algebras. I. Foundations. *J. Amer. Math. Soc.*, 15(2):497–529 (electronic), 2002.
- [G] P. Gabriel. The universal cover of a representation finite algebra *Lecture Notes in Math.* 903 (1981), 68–105.
- [GLS] Christof Geiß, Bernard Leclerc, and Jan Schröer. Rigid modules over preprojective algebras *Invent. Math.* 165 (2006), no. 3, 589–632.
- [HRS] D. Happel, I. Reiten and S. O. Smalø. Tilting in abelian categories and quasitilted algebras, *Mem. Amer. Math. Soc.* 575 (1996).
- [HU] Dieter Happel and Luise Unger. On the set of tilting objects in hereditary categories. Representations of algebras and related topics. Amer. Math. Soc., *Fields Inst. Commun.* 45 (2005), 141–159.
- [H1] Thomas Hübner. Exzeptionelle Vektorbündel und Reflektionen an Kippgarben über Projektiven Gewichteten Kurven. *Dissertation zur Erlangung des Doktorgrades des Fachbereichs Mathematik-Informatik der Universität-Gesamthochschule Paderborn*, 1996.
- [H2] Thomas Hübner. Reflections and almost concealed canonical algebras. *Sonderforschungsbereich 343, Diskrete Strukturen in der Mathematik* Universität Bielefeld, 1997.

- [I] Osamu Iyama. Higher dimensional Auslander-Reiten theory on maximal orthogonal subcategories. *Adv. Math.* 210 (2007), no. 1, 22–50.
- [IY] Osamu Iyama and Yuji Yoshino. Mutation in triangulated categories and rigid Cohen-Macaulay modules. *Invent. Math.* 172 (2008), no. 1, 117–168.
- [K] B. Keller. On Triangulated orbit categories. *Doc. Math.* 10 (2005), 551–581.
- [KR] B. Keller and I. Reiten. Acyclic Calabi-Yau categories, with an appendix by Michel Van den Bergh. *Compos. Math.* 144 (2008), no. 5, 1332–1348.
- [P] Yann Palu. Grothendieck group and generalized mutation rule for 2-Calabi-Yau triangulated categories. *J. Pure Appl. Algebra* 213 (2009) pp. 1438–1449.
- [R] Claus Michael Ringel. The self-injective cluster-tilted algebras. *Arch. Math. (Basel)* 91 (2008), no. 3, 218–225.
- [S] Ralf Schiffler. A geometric model for cluster categories of type D_n . *J. Algebraic Combin.* 27 (2008), no. 1, 1–21.
- [V] Dagfinn Vatne. The mutation class of D_n quivers. *Comm. Algebra*, 38 (2010), no. 3, 1137–1146.
- [XZ] J. Xiao, Bin Zhu. Locally finite triangulated categories. *J. Algebra* 290 (2005), no. 2, 473–490.

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