Splitting methods for oscillatory linear non-autonomous differential equations

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A usual strategy to derive numerical approximation methods for certain classes of time-dependent partial differential equations (PDEs) involve two stages. First, the space operators are discretized on an appropriate domain. This leads to a initial value problem defined by a system of ordinary differential equations (ODEs). In a second stage, a numerical scheme is chosen to carry out the integration of this ODE system, thus rendering a fully discrete integration scheme for the original PDE.

When space discretization is applied to PDEs modeling relevant physical problems (such as wave-packets dynamics in quantum mechanics, time dependent Maxwell equations in electrodynamics or some models in quantum computing), the resulting time-continuous system of ODEs is linear and has the following particular structure

$$x' = M(t)y, \qquad y' = -N(t)x. \tag{1}$$

Here $x(t) \in \mathbb{R}^{d_1}$, $y(t) \in \mathbb{R}^{d_2}$, and the matrices $M(t) \in \mathbb{R}^{d_1 \times d_2}$, $N(t) \in \mathbb{R}^{d_2 \times d_1}$ are such that NM and MN have non-negative eigenvalues. The numerical treatment of some linear parabolic PDEs also leads to systems of the form

$$x'' + D^2(t)x = 0, \tag{2}$$

with $x(t_0) = x_0, x'(t_0) = x'_0 \in \mathbb{R}^d$ and $D(t) \in \mathbb{R}^{d \times d}$.

In this work we are concerned with the second stage in the above numerical integration designing process, by proposing and analyzing new families of splitting methods for equations (1) and (2).

The strategy for constructing such methods comprises itself two different phases. In the first one, we build splitting schemes with high accuracy and large stability intervals for (1) and (2) when there is no explicit time dependency in the equations. In the second phase, these methods are adapted to the non-autonomous case by appropriately averaging the time dependency in each stage of the scheme. The resulting numerical methods are shown to be more efficient than other standard schemes on some numerical examples.