Towards the implementation of multisymplectic numerical methods for Maxwell's equations

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Many conservative PDEs (e.g. KdV equation, wave equation, Schrödinger equation) have multisymplectic formulations. This multisymplectic formulation provides another way of studying Hamiltonian PDEs that is different from the traditional view in symplectic numerical integration. Since the seminal work of Bridges and Reich [1], many theoretical and numerical studies for different conservative PDEs can be found in the multisymplectic literature. However, most multisymplectic studies focus on deriving and implementing methods for one-dimensional PDEs, and not many have investigated the practical generalization of multisymplectic methods for higher dimensional PDEs.

In this talk we give a preliminary study of applying multisymplectic methods to a higher dimensional PDE. One natural candidate is Maxwell's equations in two dimensions. Frank [2] and Su et al. [3] independently mentioned that Maxwell's equations in simple medium can be rewritten as a linear multisymplectic system with no extra transformations (i.e. no extra variables). This reduces the possibility of numerical artifacts and lowers overall computational cost.

We present some theoretical and numerical observations of Maxwell's equations related to geometric numerical integration. This includes the application of symplectic and multisymplectic methods to Maxwell's equations, the related conservation laws, and dispersion properties for these numerical methods.

References

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[3] H. Su, M. Qin and R. Scherer, *A multisymplectic geometry and a multisymplectic scheme for Maxwell's equations*, Inter. J. Pure Appl. Math. **34**(1):1-17 (2007)