WAVES IN FLUIDS AND SOLIDS (WaveMaker)

Overview

In this project, we want to pool intellectual resources at three Norwegian universities—NTNU, UiO, and UiB—to make a focused effort in the development of novel numerical methods for the study of wave phenomena.

Our long-term vision is the creation of a nationwide team of researchers contributing to numerical study of nonlinear evolutionary partial differential equations. Ideally, this collaboration will even extend to a joint Nordic effort. As a first step we have sought contact with two groups in Sweden that each are working on topics related to our research. During this project, the planned exchange between the groups will take place in particular through a student exchange program. Moreover, there will be an international conference organized at one of the three Norwegian nodes.

The particular flavor of our approach—besides the common interest in wave-type equations—consists in the adaption of recent theoretical developments in the study of evolution equations to the analysis of numerical methods. Armed with these new techniques, we will attack certain fundamental problems of wave motion in fluids and solids by implementing our methods in large-scale simulations of a selected set of representative practical problems.

Moreover, we will make use of the varied expertise of the participants to make a simultaneous study of the basic governing equations of fluid and solid body mechanics with a number of simpler asymptotic model equations, each describing certain aspects of a particular problem. Special focus will be on the validation of these model equations through the use of systematic numerical studies.

1 Introduction

In many scientific endeavors, wave phenomena play a central role. In research ranging from inverse problems related to materials properties to the study of the deep ocean, the understanding of wave propagation properties can be advantageously employed. A traditional point of view has been to extract a linear wave equation with coefficients containing information about the situation at hand. Such a linear analysis gives certain predictions, but more often than not, it proves to be inadequate.

As it is well understood by now that nonlinearity is an important feature in many problems, there has been an enormous amount of research directed into the investigation of nonlinear effects. One effective way to study nonlinearity is by considering its interplay with other important characteristics of a given problem. Such a study is often conducted in the context of a simpler model equation which encapsulates the essential features of a problem. The Korteweg–deVries (KdV) equation is a prototype of such a model equation, describing the competition between nonlinear and dispersive effects in water waves. It is also the first equation for which solitary-wave solutions were found. Solitary waves have been known mostly in the context of fluid mechanics, but have also been indicated in the study of solids. In particular, solitary waves have been useful in parameter identification problems in solids [10]. Other important examples of model equations are the kinetic equations of wave turbulence [11], and the cubic Schrödinger equation, which appears in a variety of contexts [29]. An example of a model with dissipation is given by the Burgers-KdV equation which has been recently studied by Chorin [5] in the context of turbulence.

This project contains two major themes. Firstly, an effort will be made to exploit recent advances in the theoretical study of evolution problems in the context of numerical schemes. The rapid advancement of computer technology has made it ever more possible to solve large-scale problems by means of heavy computation. The mathematical understanding of the schemes used often lags their practical implementation. Here, we want to take a measured approach to the numerical approximation of evolution equations, focusing on issues about convergence and efficiency. The question of convergence is often neglected in computer simulations. We want to use our mathematical expertise to develop methods that are guaranteed to converge. Moreover, we will be interested in algorithms that exploit features of the problem in an intelligent way, so as to solve a problem in the most efficient fashion.

Secondly, we will focus on the investigation of the validity of different model equations as approximations of the basic governing equations in fluid and solid body mechanics. In particular, it is our purpose to investigate the interplay between asymptotic model equation with the full Euler, Navier–Stokes equations, and the equations of elasticity. These are delicate issues, and our collective understanding of this question is just beginning to be formed. There exist few mathematical results in this direction, among others the work of Craig [6] on the validity of the Boussinesq equations as a model for water waves and the work of Illner [22] on the Boltzmann equation as a general model for continuum mechanics. These lines of research are mostly relying on analytical techniques, without giving much attention to numerical results. We want to study similar questions from a more practical point of view, with large-scale numerical simulations as the centerpiece.

The project consists of three main components. The first component is directed towards waves in fluids, both in the setting of viscosity dominated flows, and in the case of very low dissipation. The second component is focused on novel numerical techniques for the treatment of nonlinear wave equations which arise in this study of variational problems. Thirdly, there will be research directed towards the development of structure-preserving numerical methods for evolution equations.

2 Waves in fluids

The (incompressible and compressible) Navier–Stokes (NS) equations are at the very foundation of fluid mechanics. In this context, we are interested in two types of problems. Firstly, the focus will be on the multidimensional compressible isentropic Navier–Stokes equations. The second topic is the study of incompressible flow with and without dissipation, but in the presence of a free boundary at the surface.

2.1 Compressible Navier–Stokes equations

The multidimensional compressible isentropic Navier-Stokes equations are given by

$$\partial_t \varrho + \operatorname{div}(\varrho u) = 0, \qquad \partial_t(\varrho u) + \operatorname{div}(\varrho u \otimes u) + \nabla p = \mu \Delta u + (\lambda + \mu)\nabla \operatorname{div} u, \tag{1}$$

where the unknowns are the density $\varrho \geq 0$ and the velocity $u = (u_1, \ldots, u_N)^\top$ of the fluid, $(t, x) \in (0, T) \times \Omega$, $T > 0$, $\Omega \subset \mathbb{R}^N$, $N \geq 2$. Here, the pressure p is assumed to be given as a nonlinear function of the density (typically $p(\varrho) = a\varrho^\gamma$ for some constants $a > 0$ and $\gamma > 1$), and λ, μ are the Lamé viscosity (constant) coefficients of the fluid.

The mathematical theory, including the existence of global solutions with large discontinuous data, has advanced only recently for the compressible NS equations. The first proof of global-in-time existence for weak solutions is due to Lions and can be found in his book [25] from 1998. Some extensions of Lions' result can be found in [12]. Roughly speaking, existence is now known under the assumption that $p(\varrho) = a\varrho^\gamma$ with $\gamma > N/2$. Important ingredients in Lions' existence proof are the proof of an improved pressure estimate and the strong convergence of the density. For the latter, a new sophisticated compactness machinery (similar to the compensated compactness theories) has been developed by which weak limits can be proved to be strong. Some key steps are a suitable rewriting of the NS system so as to make a certain commutator appear, and the proof of a weak continuity property of the product of arbitrary nonlinear functions $\eta(\varrho)$ by the effective viscous flux $(\lambda + 2\mu)\operatorname{div} u - p(\varrho)$.

In the field of computational fluid mechanics, many numerical approaches have been developed for computing compressible flows with negligible and non-negligible viscous effects. Popular discretization schemes are Godunov type finite difference/finite volume methods based on Riemann solvers, and finite element methods including continuous as well as discontinuous shape functions, the latter giving rise to the the so-called discontinuous Galerkin method. The discontinuous Galerkin method has become very popular in recent years for computing shock wave type solutions of the compressible NS system and the corresponding conservation laws. The discontinuous finite elements methods offer a unified framework for development and analysis of discretizations for different flow models based on their variational formulations. Moreover, other known finite volume methods may often be viewed as special cases. During the 1980s and 1990s, large efforts were put into the development of convergence theory for first and second order numerical methods for scalar conservation laws in multiple dimensions, using first *bounded total variation* compactness tools and later using more sophisticated tools like compensated compactness and measure valued solutions (Murat-Tartar, DiPerna, Szepessy, Cockburn, LeFloch, etc.), see [27] for an overview. In the case of 2×2 systems of conservation laws in one spatial dimension, one can use the compensated compactness method to obtain such a convergence theory (DiPerna, Chen, Lions-Souganidis-Perthame, etc.).

Regarding the above multi-dimensional NS system (1) very little is known about convergence theory for numerical methods (but see Lions' book [25] for a time discretization scheme and some work by Giacomoni and Orenga (2003) for a special Galerkin scheme for a two-dimensional problem with $\gamma = 2$, and various papers by Hoff, Chen, Trivisa, etc., for one-dimensional systems). The new compactness machinery of Lions gives hope to change this unfortunate situation. Indeed, the goal of this part of the project is develop convergence theory for at least some numerical methods for computing solutions of (1) by translating Lions' compactness machinery to a discrete or semi-discrete setting. A long-term vision is to have a convergence theory for numerical methods such as the first order discontinuous Galerkin method for the full system (1). However, we will start to develop such a theory for simpler multidimensional variants of the NS system.

2.2 Free surface flows

In this second part, we consider the simulation of free surface waves as described by the incompressible Navier-Stokes or Euler equations in time-dependent geometries, and with the appropriate free surface boundary conditions imposed. Special emphasis is given to the effect of surface tension T. One popular measure of capillarity is the Bond number $\tau = T/(\rho g h^2)$, where g is the gravitational constant, ρ is the (constant) density of the fluid, and h is the undisturbed height. In particular, if τ exceeds $\frac{1}{3}$ it is commonly expected that capillarity will be dominating gravitational effects. This is displayed effectively in the corresponding KdV equation

$$\eta_t + \eta\eta_x + \left(\tfrac{1}{3} - \tau\right)\eta_{xxx} = 0, \tag{2}$$

whose solitary waves are of depression when τ is greater than $\frac{1}{3}$. A study of the KdV and related model equations, such as a system of coupled KdV equations and the KdV-Burgers equation will aid us in identifying interesting parameter ranges for numerical simulation of the full equations Euler or NS equations. For model equations such as KdV, the main emphasis will be on gaining a better understanding of the spectral convergence of Fourier Galerkin and collocation methods. In particular, we will attempt to make use of recent analytical work in [2] to improve convergence estimates of some spectral schemes.

In numerical implementations of approximating schemes for the full Euler or NS equations with a free boundary, high-order spectral element methods come to the fore. In the time stepping, the update of the geometry is typically done separately from the interior NS calculations. Here we will follow an Arbitrary Lagrangian Eulerian (ALE) formulation [16], which follows that general pattern. Surface tension is typically important at smaller length scales and acts as a stabilizing force. Surface tension effects can give rise to capillary waves, which are short free surface waves. When simulating surface tension dominated problems, an explicit update of the free surface gives rise to a time step restriction that can be much more severe than the Courant condition associated with an explicit treatment of the nonlinear convection term [17]. On the other hand, when surface tension effects are negligible, an explicit free surface update may be unstable. We propose to investigate free surface problems over the entire range of surface tension forces with the aim towards constructing higher order splitting schemes that are both robust and computationally efficient.

Another option for the space discretization is the use of reduced basis methods. Reduced basis methods have shown promise as a way to construct low-dimensional models of physical systems; such models are attractive to use in optimization and real time control, and as a way to solve complex problems. Significant work in this direction has been done in the context of both steady and unsteady problem [28 26]. We remark that reduced basis methods may be viewed as a particular compression technique which may be exploited in numerical simulations. We propose to investigate this computational framework for simulating wave propagation problems. Very few results are currently available for such problems. This work will primarily be done in the context of solving simpler model problems.

3 Wave equations of variational type

Nonlinear wave equations frequently appear in the study of variational problems in solid body mechanics, and in the study of fluids. Here we will focus on numerical techniques that exploit the special variational structure of such problems.

3.1 Conservative methods

In many situations, variational wave equations arise as the Euler–Lagrange equation of a certain variational principle. For example, the Euler–Lagrange equations for the action

$$\frac{\delta}{\delta u} \int \left(u_t^2 - c^2(u) \left|\nabla u\right|^2 \right) dxdt = 0,$$

leads to the nonlinear wave equation

$$u_{tt} - c^2(u)\Delta u = c(u)c'(u) \left|\nabla u\right|^2 . \tag{3}$$

Here u is real valued a function on $\mathbb{R}^d \times \mathbb{R}^+$, where $d = 1$, 2 or 3. Such equations arise in a variety of models, ranging from general relativity to waves in liquid crystals. In one space dimension, i.e., $d = 1$, existence of a weak solution to the Cauchy problem has been established by Hunter, Zhang and Zheng [21 31]. These papers also contain some partial uniqueness statements, but the best result so far in this direction was obtained by Bressan, Zhang, and Zheng [3]. Concerning computations, there has been little activity concerning equations like (3), especially with regard to convergence of numerical schemes. Some simple schemes for an asymptotic equation related to (3) have been treated in [18].

We are pursuing a line of research which investigates numerical methods for nonlinear wave equations, both in one space dimension and several space dimensions. One can define a general class of schemes for (3) based on similar methods for hyperbolic conservation laws. In fact, by using and modifying modern approximation schemes for conservation laws, one would expect to get suitable discretizations for a class of nonlinear wave equations like (3). This remains to be tested, and if successful, compared with available experimental results. Finally, we will pursue the natural question of how schemes based on conservation laws compare with nonconservative schemes based directly on the equation.

A natural extension of this project would be to establish the theoretical convergence on numerical schemes for various classes of initial data. Such an agenda might be in reach only in the special case of of one space dimension. Nevertheless, this convergence will have practical interest, since multidimensional generalizations of convergent one-dimensional schemes tend to produce more trustworthy results.

3.2 Variational inequalities

In some situations, nonlinear wave equations arise from variational inequalities. For example, the study of crack and impact mechanisms involve contact problems described by variational inequalities. Another interesting scientific application is in the study of glaciers. Here we want to exploit the special variational structure of these problems to develop numerical techniques which are better adapted to the problems at hand than the usual finite-difference or spectral methods used for wave-type equations. In particular, we want to study the numerical treatment based on efficient Finite Element (FE) schemes for second-order evolution contact problems. One example of such a scheme is the so-called discontinuous Galerkin method.

The study of the impact load transfer in mechanical assemblages are of considerable importance in a wide range of engineering applications. In one of the first contributions to the subject Hughes and collaborators emphasized the wave propagation aspects of the problem. As a monograph on the corresponding numerical treatment we mention Hughes [20]. A prototypical example is the so-called dynamic obstacle problem, i.e., the scalar deflection u and the corresponding stress field $\sigma = (\sigma_1, \sigma_2)$ of an elastic membrane under time-dependent load f have to be determined on a bounded domain $\Omega \subset \mathbb{R}^2$ subjected to certain constraints. Eventually this dynamic contact problem has to fulfill the following equations

$$\begin{aligned} \dot{v} - \operatorname{div} \sigma = f, \qquad & \sigma = \nabla u, \\ u(0) = u_0, \dot{u}(0) = v_0, \quad & u = 0 \text{ on } \partial\Omega, \end{aligned}$$

and the kinematic conditions $(-\operatorname{div} \sigma - f)(u - \psi) = 0$ and $(-\operatorname{div} \sigma - f)(\dot{u} - \dot{\psi}) = 0$. Setting $a(.,.) = (\nabla ., \nabla .)$, the adequate mathematical formulation (see, e.g., Kikuchi and Oden [24]) seeks a variational solution u with $u(0) = u_0$, $\dot{u}(0) = v_0$ in $\dot{K} \subset V$ with $V := W^{2,\infty}(I; L^2(\Omega)) \cap W^{1,\infty}(I; H_0^1(\Omega))$, and $\dot{K}$ additionally fulfilling the kinematic conditions, satisfying the inequality setting

$$(\partial_t^2 u, \dot{\varphi} - \dot{u}) + a(u, \dot{\varphi} - \dot{u}) \geq (f, \dot{\varphi} - \dot{u}), \quad \dot{\varphi} \in \dot{K}. \tag{4}$$

The main goal is to develop efficient and robust numerical methods to treat variational inequalities having the above structure. To this end, a number of issues will be addressed. With the aim of having unconditionally stable time-integration schemes, implicit schemes are a must for the impact problems under consideration. In order to analyze the discretization within a general Galerkin framework, discontinuous Galerkin schemes in time are appropriate. We are developing and investigating new schemes with respect to energy and momentum conservation. Additional care is needed to avoid numerical oscillations [8].

At each time step we need a robust discretization in space. Based on the knowledge gained for stationary problems (see, e.g., [15 30]) we investigate, how such discretizations can be improved and applied to the time-dependent contact problems arising in this project. Also at each time step one has to solve a large sparse systems with inequality constraints. So there is a natural interest to solve the inequalities which arise from a finite element discretization by efficient iterative schemes. We plan to improve the results of the studies begun in [1].

We plan to apply our new methods to real-life problems in multi-physics, where impact problems built an important part in simulating e.g. complex high-speed-forming processes with friction and lubrication.

4 Hamiltonian models and geometric time integration

This last component is concerned with special numerical techniques for partial differential equations of Hamiltonian form. For conservative partial differential equations like various wave equations it is often of interest to perform simulations over long times. It is generally believed that integration schemes should in some sense inherit the conservative properties of the continuous system in order to reproduce the correct structural behavior of the exact solution. Integrators designed for this purpose are called geometric integrators. In ordinary differential equations, it is well known that integrators which conserve the symplectic or reversible structure of the Hamiltonian problem have many favorable properties [14] which can be explained through the use of Backward Error Analysis and KAM theory. The perhaps most obvious approach for generalizing these ideas to partial differential equations is to discretize the Hamiltonian in space and then apply a symplectic integrator to the resultant ordinary differential equation system. For completely integrable partial differential equations, one may even look for completely integrable discretization, as the Ablowitz-Ladik model for the nonlinear Schrödinger equation, or the Toda lattice for the KdV equation (2). However, being interested in the long time behavior, it is crucial that the time step can be taken relatively large. This may be difficult to reconcile with the strong non-resonant condition known from KAM theory when the number of spatial degrees of freedom becomes large. One way of understanding the shortcomings of such an approach is through the observation that the symplectic structure for the partial differential equation is global in space, such that the preservation of symplecticity is in some sense averaged in space and therefore it does not represent a sufficiently strong criterion for structure preservation. Bridges and Reich [4] introduce the concept of a multisymplectic formulation of a Hamiltonian partial differential equation. Along with such a formulation comes a local symplectic structure from which local conservation laws of energy and momentum can be derived. A multisymplectic integrator is a numerical scheme which satisfies a discrete version of the multisymplectic conservation law. Several multisymplectic schemes are known, in particular among the subclasses of finite difference schemes, finite volume schemes, finite element methods, and spectral discretization. Extensive numerical tests have been performed with such schemes, showing that local conservation is well preserved (though not exactly) over long times. Attempts have been made to explain this benign behavior through the use of backward error analysis, but as of now, only partial results are known for some selected Hamiltonian equations. A drawback with many of the multisymplectic integrators is that they are fairly expensive, and the associated nonlinear algebraic systems need to be solved to machine accuracy in order not to destroy the geometric properties. Nevertheless some success has been obtained in constructing semi-explicit multisymplectic schemes, and there is a multitude of interesting issues which we plan to investigate as part of this project.

One obvious task is to conduct numerical tests with symplectic and multisymplectic discretizations of some selected Hamiltonian partial differential equations, possibly in connection with new finite element based space discretizations. This includes the comparison with globally conservative schemes on counts of long-time behavior and computational cost.

Another issue is the applications of backward error analysis for such schemes. This encompasses the search for approximate versions of local energy and momentum conservation laws which are exactly conserved by the numerical scheme.

It will also be important to search for schemes with less computational cost, but with the same conservative properties. One may also try to find semi-explicit schemes, and Lie group or exponential type integrators for semilinear partial differential equations with favorable properties.

One aspect of our work will be the application of these techniques to Hamiltonian model equations. In particular, it will be interesting to apply these techniques to some new Hamiltonian equations for the evolution of internal waves which were recently found in [7]. Some model equations for waves in elastic rods are also of Hamiltonian form. In particular, the Camassa-Holm equation has been indicated in this context [9]. It will be interesting to analyze rapidly convergent and other fast numerical methods for such equations in connection with symplectic time integrating schemes [23].

5 Participants

Helge Holden, NTNU, Project leader

Helge Holden (Dr.philos., University of Oslo, 1985) is Professor of mathematics at the Norwegian University of Science and Technology (NTNU). After one year as a Postdoctoral Fellow at Courant Institute, New York University, he accepted a position at NTNU, where he from 1991 has been a full professor. He has published more than 110 papers, and written 5 books. His area of interest is differential equations, ranging from theory and numerical methods to applications. Applications are mostly focused on flow in porous media. He has supervised 14 PhD students and is currently supervising 4 PhD students. He has been the principal investigator of several projects funded by the Research Council of Norway, and has actively participated in several projects with both domestic and international funding. Currently he is President of ECMI (European Consortium of Mathematics in Industry) and Secretary of the European Mathematical Society.

Henrik Kalisch, UiB

Henrik Kalisch is Associate Professor in hydrodynamics at the Department of Mathematics at Bergen University, starting Fall 2005. He received his PhD in 2001 from the University of Texas at Austin. Prior to arriving in Bergen he held postdoctoral positions at NTNU, Lund University, Sweden and McMaster University, Canada. He was also a summer intern at Los Alamos National Laboratory, USA. His research is focused on mathematical problems in fluid mechanics, mathematical modeling in the physical sciences, and fast and rapidly convergent numerical methods for evolution equations. He has published more than 10 articles in international journals.

Kenneth Hvistendahl Karlsen, UiO

Kenneth H. Karlsen is Professor of mathematics at Centre of Mathematics for Applications/University of Oslo (since 2004). He received his Dr.scient. degree from the University of Bergen (UiB) in 1998. After one year as a Postdoctoral Fellow at UiB, he accepted a position at UIB, where he worked until 2004, from 2001 as a full professor. He has published more than 100 papers. His area of interest is nonlinear partial differential equations, ranging from theory and numerical methods to applications. Applications are mostly focused on flow in porous media and solid-liquid separation processes as well as mathematical finance. He received the 1998 Meltzer Price for Young Researchers at University of Bergen and an Outstanding Young Investigators Award (YFF), Research Council of Norway 2004. He has co-supervised 3 PhD students and 9 master students, and is currently co-supervising 3 PhD students. He has been involved in several projects funded by the Research Council of Norway and projects with both domestic and international funding.

Nils Henrik Risebro, UiO

Nils Henrik Risebro is Professor of mathematics at the Centre of Mathematics for Applications/University of Oslo. He received his Dr. Scient. degree from the University of Oslo in 1991. From 1991 to 1994 he held a postdoctoral position at the University of Oslo. In 1994 he obtained a tenured position at the University of Oslo, where he has been since, from 2002 as a full professor. He has published more than 50 papers and co-written one book. His area of interest is nonlinear partial differential equations, mainly equations of hyperbolic type. He has supervised 3 PhD students, and more than 15 master students. He is currently supervising one PhD student.

Brynjulf Owren, NTNU

Brynjulf Owren is a Professor of Mathematical Sciences at NTNU. He was employed in SERES A/S from 1985 to 1988 where he was doing research on marine seismic sources. He received his Dr. Ing. degree from NTNU in 1990 and was a postdoc at the University of Toronto from 1990 to 1992. From 1994 he got a tenured position at NTNU where he has been since, from 1999 as a full professor. He has published more than 30 papers, and has supervised 6 PhD students. He has been the principal investigator in projects funded by the Research Council of Norway, he is employed part time by Sintef as a consultant, and is currently head of the NTNU student program in industrial mathematics. In 2002-2003 he was heading a research group in Geometric Integration at the Center for Advanced Study in Oslo. His area of interest is within numerical solution of differential equations and geometric integration.

Einar Rønquist, NTNU

Einar M. Rønquist is Professor of mathematics at the Norwegian University of Science and Technology (NTNU). He holds a PhD from Massachusetts Institute of Technology in 1988. Previous appointments include Postdoctoral Fellow at M.I.T. (1988-1989), Vice President of R&D at Nektonics, Inc. (1991-1999), Lecturer at M.I.T. (part time, 1999), Professor at NTNU since 1999. Areas of interest are computational methods for partial differential equations, in particular, pertaining to fluid dynamics applications: spectral and finite element methods, domain decomposition methods, reduced basis methods. He has published more than 30 articles in scientific and engineering computation, and is currently supervising two PhD students. He serves as a Leader of Research Program in Computational Science and Visualization at NTNU.

Franz-Theo Suttmeier, NTNU

Franz-Theo Suttmeier has been an Associate Professor of mathematics at NTNU from January 2005.. He received his Dr. rer. nat. degree from the University of Heidelberg in 1997. As a postdoc he finished his Professorial dissertation (Habilitation) at the University of Dortmund in 2002, and accepted his current position at NTNU in January 2005. He has published about 20 refereed articles and one book (to appear 2005). He co-supervised 2 and currently supervises 2 PhD students. His work is mostly motivated by industrial applications, in particular contact problems requiring efficient, adaptive finite elements for variational inequalities.

6 Structure of the project

This project will run for four years. During this time, it is planned to fund two PhD students for three years each, a postdoctoral fellow for a two-year period, and a postdoctoral fellow for a one-year period. The workplace of the graduate students and postdoctoral fellows will depend on the affiliation of their respective supervisor.

7 Collaboration

It is our intention to collaborate with two separate groups in Sweden. Professor Peter Hansbo of Chalmers University is already involved in joint research with Franz-Theo Suttmeier. A second collaboration will be initiated with the group of Björn Engquist at KTH in Stockholm, and Per Lötstedt at Uppsala University. For the graduate students, it will be possible to spend one semester in Sweden, either at Chalmers University in Göteborg, at Uppsala University, or at KTH.

If our Swedish counterparts will obtain funding, they in turn promise to send graduate students hired in connection with this project to one of the Norwegian nodes. Besides this student exchange, we aim to organize an international conference during the third or fourth year of the project. The goal of such a conference is to increase visibility of our research group on an international level, to extend relations with our Swedish and trans-Atlantic collaborators, and to further the cause of a Nordic initiative for the numerical treatment of wave phenomena.

References

[1] H. Blum, D. Braess, and F.T. Suttmeier. *A cascadic multigrid algorithm for variational inequalities*, Computing and Visualization in Science **7** (2004), 153-157.

[2] J.L. Bona, Z. Grujić and H. Kalisch, *Algebraic lower bounds for the uniform radius of spatial analyticity for the generalized KdV equation*, to appear in Ann. Inst. H. Poincaré, Anal. Non Linéaire.

[3] A. Bressan, P. Zhang, and Y. Zheng, *On asymptotic variational wave equations*, to appear in Arch. Rat. Mech. Anal.

[4] T.J. Bridges and S.Reich, *Multi-symplectic integrators: numerical schemes for Hamiltonian PDEs that conserve symplecticity*, Phys. Lett. A **284** (2001), 184-193.

[5] A.J. Chorin, *Averaging and renormalization for the Korteweg-deVries-Burgers equation*, Proc. Natl. Acad. Sci. USA **100** (2003), 9674-9679.

[6] W. Craig, *An existence theory for water waves and the Boussinesq and Korteweg-deVries scaling limits*, Comm. Partial Differential Equations **10** (1985), 787-1003.

[7] W. Craig, P. Guyenne and H. Kalisch, *Hamiltonian long-wave expansions for free surfaces and interfaces*, to appear in Comm. Pure and Appl. Math.

[8] A. Czekanski and S.S. Meguid, *Analysis of dynamic frictional contact problems using variational inequalities*, Finite Elements in Analysis and Design **37** (2001), 861-879.

[9] H.-H. Dai and Y. Huo, *Solitary shock waves and other travelling waves in a general compressible hyperelastic rod*, Proc. Roy. Soc. London A **456** (2000), 331-363.

[10] V.I. Erofeev, *Wave Processes in Solids with Microstructure*, World Scientific, Singapore, 2003.

[11] G.C. Falkovich, V.S. L'vov and V. E. Zakharov, *Kolmogorov Spectra of Turbulence. I. Wave Turbulence*, *Series in nonlinear dynamics*, Springer-Verlag, Berlin, 1992.

[12] E. Feireisl, *Viscous and/or heat conducting compressible fluids*, in Handbook of mathematical fluid dynamics, Vol. I, pp. 307-371, North-Holland, Amsterdam, 2002.

[13] O. Gonzalez, *Exact Energy-Momentum Conserving Algorithms for General Models in Nonlinear Elasticity*, Comp. Meth. Appl. Mech. Eng., **190** (2000), 1763-1783.

[14] E. Hairer, C. Lubich, and G. Wanner, *Geometric Numerical Integration*, Springer Series in Computational Mathematics. Springer-Verlag, Berlin, 2002.

[15] P. Hansbo and P. Heintz, *Stabilized Lagrange multiplier methods for elastic contact with friction*, Technical report, Chalmers Finite Element Center, 2004; www.phi.chalmers.se/preprints.

[16] L.-W. Ho and A.T. Patera. *A Legendre spectral element method for simulation of unsteady incompressible viscous free-surface flows*, Comp. Meth. Appl. Mech. Eng. **80** (1990) 355-366.

[17] L.-W. Ho and E.M. Rønquist, *Spectral element solution of steady incompressible viscous free-surface flows*, Finite elements in analysis and design **16** (1994) 207-227.

[18] H. Holden, K.H. Karlsen, and N.H. Risebro, *Numerical schemes for the Hunter–Saxton equation*, in preparation.

[19] H. Holden, X. Raynaud, *A convergent numerical scheme for the Camassa–Holm equation based on multipeakons*, to appear in Disc. Cont. Dyn. Syst.

[20] T.J.R. Hughes, *The finite element method: linear static and dynamic finite element analysis*, Englewood Cliffs, NJ, Prentice-Hall, 1987.

[21] J.K. Hunter and Y. Zheng, *On a nonlinear hyperbolic variational equation: II. The zero-viscosity and dispersion limits*, Arch. Rat. Mech. Anal. **129** (1995), 355-383.

[22] R. Illner, *Derivation and validity of the Boltzmann equation: some remarks on reversibility concepts, the H-functional and coarse-graining*, Material instabilities in continuum mechanics (Edinburgh, 1985–1986), 147–174, Oxford Sci. Publ., Oxford Univ. Press, New York, 1988.

[23] R. Kalisch and X Raynaud, *Convergence of a Spectral Projection of the Camassa–Holm Equation*, submitted.

[24] N. Kikuchi and J.T. Oden. *Contact Problems in Elasticity: A Study of Variational Inequalities and Finite Element Methods*, SIAM Stud. in Appl. Math. **8** (1988).

[25] P.-L. Lions, *Mathematical topics in fluid mechanics. Vol. 2.* The Clarendon Press Oxford University Press, New York, 1998.

[26] A.E. Løvgren, Y. Maday, and E.M. Rønquist, *A reduced basis element method for the steady Stokes problem*, submitted.

[27] J. Málek, J. Nečas, M. Rokyta, and M. Růžička, *Weak and measure-valued solutions to evolutionary PDEs.* Chapman & Hall, London, 1996.

[28] C. Prud'homme, D.V. Rovas, K. Veroy, L. Machiels, Y. Maday, A.T. Patera, and G. Turinici, *Reliable real-time solution of parametrized partial differential equations: Reduced basis output bound methods.* J. Fluids Engineering **124** (2002) 81-90.

[29] C. Sulem and P.L. Sulem, *The Nonlinear Schrödinger Equation*, Series in applied mathematics, Springer-Verlag, Berlin, 1999.

[30] F.T. Suttmeier, *On computational methods for variational inequalities*, Comp. Mech. **2005**,

[31] P. Zhang and Y. Zheng, *Weak solutions to a nonlinear variational wave equation*, Arch. Rat. Mech. Anal. **166** (2003), 303-319.