On the Riemann Problem for a Hyperbolic Two-Phase Flow Model

D. Zeidan[a,*], E. Toro[b] and A. Slaouti[c]

[a]Department of Computing and Mathematics
Manchester Metropolitan University, Manchester M1 5GD, UK

[b]Department of Civil and Environmental Engineering
University of Trento, 38050 Trento, Italy

[c]Department of Engineering and Technology
Manchester Metropolitan University, Manchester M1 5GD, UK

The two-phase flow problems in this paper describe one-dimensional isentropic mixtures of liquid and vapour, such as those occuring in nuclear reactors cooled by water under pressure. The model consists of equations for the conservation of mass, momentum of the mixture and the relative velocity between the two-phases [1, 2]. It is shown [3] that the system is strictly hyperbolic provided that the relative velocity between the two phases is much lower than the speed of sound of the two-phase mixture. A procedure is then developed for solving the Riemann problem for a mixture of liquid and vapour obeying an equation of state of the form $\rho = \rho(\mathcal{P})$ where ρ and $\mathcal{P}$ are the mixture density and pressure, respectively. An exact Riemann solver and several approximate Riemann solvers for the simplified two-fluid model are presented. The main step in solving the Riemann problem is to find the constant state $\mathbb{W}^\star$ in the star region. This depends on appropriate relations through the left and right waves connecting $\mathbb{W}^\star$ to the states $\mathbb{W}_\mathfrak{L}$ and $\mathbb{W}_\mathfrak{R}$ respectively. However, under certain extreme flow conditions, it is recommended that a combination of the exact and approximate solvers for the Riemann problem be employed. As long as the simplified two-fluid model is hyperbolic, Godunov-type numerical methods for the model can be employed using locally approximate analytical solution to the wave propagation equations. Second-order extensions of Godunov-type schemes, as well as the emergence of TVD algorithms have brought about the possibility to capture the discontinuities with very good accuracy as achieved using MUSCL-Hancock and WAF [3, 4] schemes. The numerical results for two-phase shock tube test problems closely match the results obtained by the exact solution of the Riemann problem. The tests are used to illustrate some typical wave patterns resulting from the solution of the Riemann problem. For comparison we also show numerical results obtained by the Lax-Friedrichs method. These numerical methods, which are widely used for fluid dynamic calculations, have proved to be very efficient for numerical solutions of two-phase flow problems [3].

References

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*Corresponding author. E-mail: d.zeidan@mmu.ac.uk. Tel. + 44-161-247 1500. Fax. + 44-161-247 1483.