Eksamen SIA 0003 H 2006 FASIT

1 $f(x) = (x-2)e^x$, $x \in \mathbb{R}$

a) $f'(x) = (x-2)e^x + 1\cdot e^x = (x-1)e^x$

$f''(x) = (x-1)e^x + 1\cdot e^x = x e^x$

b)

		0		1		2	
$f(x)=(x-2)e^x$	$-$		$-$		$-$		$+$
$f'(x)=(x-1)e^x$	$-$		$-$		$+$		$+$
$f''(x)=xe^x$	$-$		$+$		$+$		$+$

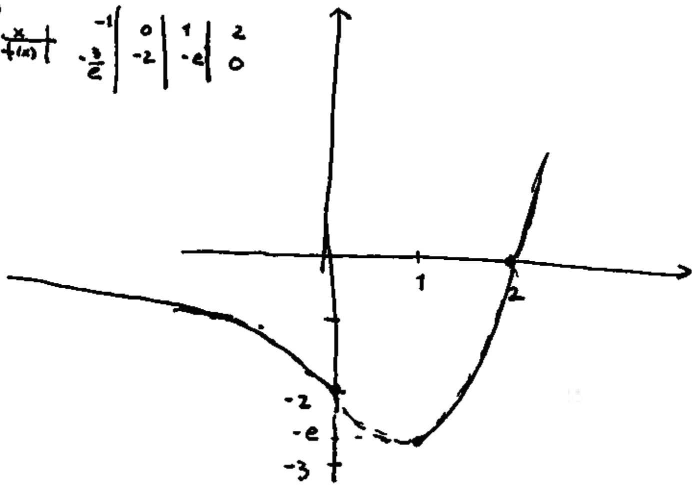
c) $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (x-2)e^x$

$= \lim_{x\to-\infty} \frac{x-2}{e^{-x}} \overset{l'Hop}{=} \lim_{x\to-\infty} \frac{1}{e^{-x}} = 0$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (x-2)e^x = \infty$

d)

x	-1	0	1	2
$f(x)$	$-\frac{3}{e}$	-2	$-e$	0



Skjæring y-aksen: $(0,-2)$

Nullpkt $x = 2$, $(2,0)$

Lokalt minpkt (globalt) $x = 1$ $(1,-e)$

Vendepunkt $x = 0$ $(0,-2)$

Horisontal asymptote $y = 0$ når $x \to -\infty$

e) $\int_0^2 (x-2)e^x\,dx = \Big|_0^2 (x-2)e^x - \int_0^2 1\cdot e^x\,dx$

$= \Big|_0^2 (x-3)e^x = (-1)e^2 - (-3)\cdot 1 = \underline{\underline{3-e^2}}$

≈ -4.389056

2 $N(t)$ antall bakterier ved tid t

$N(0) = 20$

$N(t)$ vokser eksponentielt, fordoblingstid $t = 8$

A(t1) $N(t) = 20\cdot 2^{t/8}$

A(t2) $\frac{dN}{dt} = k\cdot N \overset{N\neq 0}{\Leftrightarrow} \frac{1}{N}\,dN = k\,dt$

$\Leftrightarrow N(t) = C_0 e^{kt}$

$N(0) = 20 = C_0 \Rightarrow N(t) = 20e^{kt}$

$N(8) = 2N(0) = 20\cdot e^{k\cdot 8}$

$= 2\cdot 20$

$\Leftrightarrow e^{k8} = 2 \Leftrightarrow 8k = \ln 2$

$\Leftrightarrow k = \frac{1}{8}\ln 2$

$N(t) = 20\cdot e^{\frac{\ln 2}{8}t} = 20\left(e^{\ln 2}\right)^{\frac{t}{8}}$

$= \underline{20\cdot 2^{t/8}}$

$\underline{\underline{N(t) = 20\cdot 2^{t/8}}}$

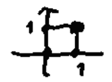
b) Bestemme t slik at

$N(t) = 1000 = 20\cdot 2^{t/8}$

$\Leftrightarrow 2^{t/8} = 50 \Leftrightarrow \ln(2^{t/8}) = \frac{t}{8}\ln 2 = \ln 50$

$\Leftrightarrow \frac{t}{8} = \frac{\ln 50}{\ln 2} \Leftrightarrow t = 8\cdot\frac{\ln 50}{\ln 2} \approx \underline{45,1}$

3 $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ Kvadrat K



a) $A^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$

$A^3 = A^2\cdot A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I\cdot A = -A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$A^4 = (A^2)^2 = (-I)^2 = I^2 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

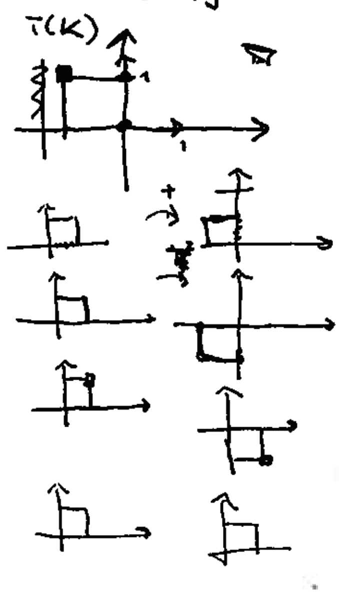
b) $T\begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$T\begin{pmatrix}1\\0\end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$

$T\begin{pmatrix}0\\1\end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-1\\0\end{bmatrix}$

$T\begin{pmatrix}1\\1\end{pmatrix} = T\begin{pmatrix}1\\0\end{pmatrix} + T\begin{pmatrix}0\\1\end{pmatrix} = \begin{bmatrix}-1\\1\end{bmatrix}$

T(K)



c) $T^2(\vec{x}) = A^2\vec{x} = -I\vec{x} = -\vec{x}$

$T^3\vec{x} = A^3\vec{x} = -A\vec{x} = -T(\vec{x})$

$T^4\vec{x} = \vec{x}$