Fasit MA 0003 V2007

1a) $\lim_{x\to 0} \frac{e^x-1}{x} = \lim_{x\to 0} \frac{e^x}{1} = 1$ (Brukar l'Hospitals regel)

b) $x-y-z=0$, $x+y=0$, $3x+y+z=4$ — linear løysing — $x=1$, $y=-1$, $z=2$

2a) $(x+3):(x-1) = 1 + \frac{4}{x-1}$ (poly.-div.)

b) $D_f = \mathbb{R}\setminus\{1\}$

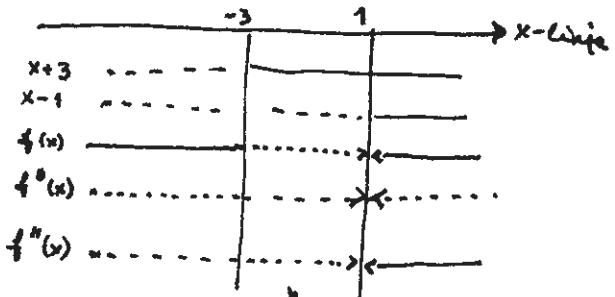
Vertikal asymptote $x=1$

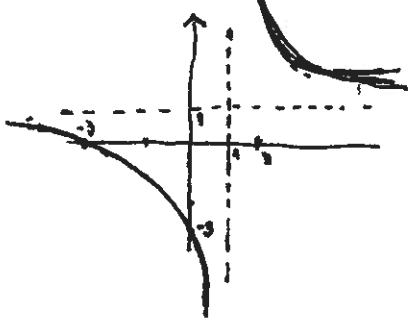
Horisontal asymptote $y=1$ ($\lim_{x\to\pm\infty} \frac{x+3}{x-1} = 1$)

$f(x) = 1 + \frac{4}{x-1}$

$f'(x) = -\frac{4}{(x-1)^2}$

$f''(x) = \frac{8}{(x-1)^3}$



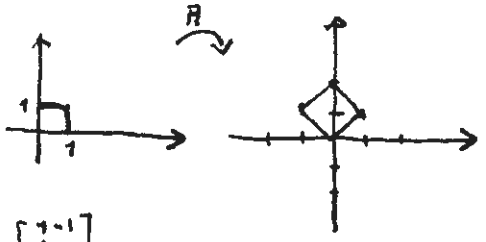


Viser til Ha fortegnsskjema

a) $\int_2^3 f(x)\,dx = \int_2^3 \{1 + \frac{4}{x-1}\}\,dx = \Big|_2^3 \{x + 4\ln|x-1|\}$

$= 1 + 4\ln 2$

3a) $A\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$, $A\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix}$, $A\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\2\end{pmatrix}$



$A = \begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$

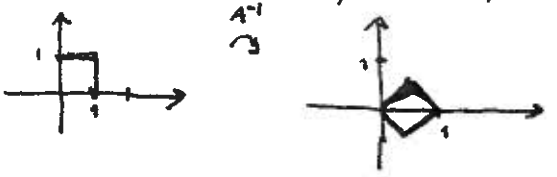
b) $A|I$, $\left(\begin{array}{cc|cc}1&-1&1&0\\1&1&0&1\end{array}\right)$ (radoperasjonar gir)

$\left(\begin{array}{cc|cc}1&0&\frac{1}{2}&\frac{1}{2}\\0&1&-\frac{1}{2}&\frac{1}{2}\end{array}\right)$

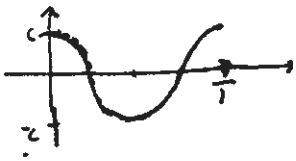
$A^{-1} = \begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2}\end{bmatrix}$

(eller $\det(A) = 2$, $A^{-1} = \frac{1}{2}\begin{bmatrix}1&-1\\1&1\end{bmatrix}^t = \frac{1}{2}\begin{bmatrix}1&-1\\-1&1\end{bmatrix}$ (kofaktormetoden))

$A^{-1}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\\-\frac{1}{2}\end{pmatrix}$, $A^{-1}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\end{pmatrix}$, $A^{-1}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$, $A^{-1}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$



4a) $v(t) = c\cos\frac{2\pi t}{T}$



b) $V(t) = V(0) + \int_0^t v(\tau)\,d\tau$

$= V(0) + \int_0^t c\cdot\cos\left(\frac{2\pi\tau}{T}\right)d\tau$ (substitusjon $u = \frac{2\pi\tau}{T}$)

$= V(0) + c\cdot\frac{T}{2\pi}\int_0^{2\pi t/T}\cos u\,du$

$= V(0) + \frac{cT}{2\pi}\Big[\sin u\Big]_0^{2\pi t/T}$

$= V(0) + \frac{cT}{2\pi}\sin\left(\frac{2\pi t}{T}\right)$

$V(6) = V(0) + \frac{2\cdot 24}{2\pi}\sin\left(\frac{2\pi\cdot 6}{24}\right)$

$\approx 47{,}6\ m^3 \approx 48\ m^3$