R.1 Graphs and Equations

OBJECTIVES

- Graph equations.
- Use the graphs as mathematical models to make predictions.
- Carry out calculations involving compound interest.

R.1 Graphs and Equations

DEFINITION:

The **graph** of an equation is a drawing that represents all ordered pairs that are solutions of the equation.

R.1 Graphs and Equations

THEOREM 1

If an amount P is invested at interest rate i, expressed as a decimal and compounded annually, in t years it will grow to an amount A given by

$$A = P(1 + i)^t.$$

R.1 Graphs and Equations

THEOREM 2

If a principal P is invested at interest rate i, expressed as a decimal and compounded n times a year, in t years it will grow to an amount A given by

$$A = P\left(1 + \frac{i}{n}\right)^{nt}.$$

R.2 Functions and Models

OBJECTIVES

- Determine whether or not a correspondence is a function.
- Find function values.
- Graph functions and determine whether or not a graph is that of a function.
- Graph functions that are piecewise defined.

R.2 Functions and Models

DEFINITION:

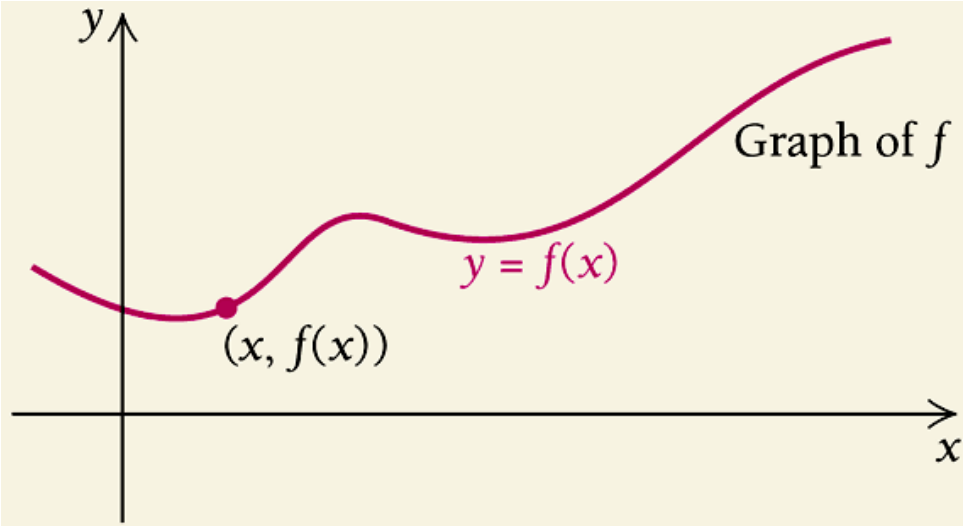
A **function** is a correspondence between a first set, called the **domain**, and a second set, called the **range**, such that each member of the domain corresponds to *exactly one* member of the range.

R.2 Functions and Models

Definition

The **graph** of a function f is a drawing that represents all the input-output pairs, $(x, f(x))$.

In cases where the function is given by an equation, the graph of a function is the graph of the equation $y = f(x)$.



R.2 Functions and Models

The Vertical Line Test

A graph represents a function if it is impossible to draw a vertical line that intersects the graph more than once.

R.3 Finding Domain and Range

OBJECTIVES

- Write interval notation for a set of points.
- Find the domain and range of a function.

R.3 Finding Domain and Range

Intervals: Notation and Graphs

Interval Notation	Set Notation	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \leq x \leq b\}$	
$[a, b)$	$\{x \mid a \leq x < b\}$	
$(a, b]$	$\{x \mid a < x \leq b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \geq a\}$	

R.3 Finding Domain and Range

Intervals: Notation and Graphs (concluded)

Interval Notation	Set Notation	Graph
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \leq b\}$	
$(-\infty, \infty)$	$\{x \mid x \text{ is a real number}\}$	

R.4 Slope and Linear Functions

OBJECTIVES

- Graph equations of the types $y = f(x) = c$ and $x = a$.
- Graph linear functions.
- Find an equation of a line when given its slope and one point on the line and when given two points on the line.
- Solve applied problems involving slope and linear functions.

R.4 Slope and Linear Functions

THEOREM 3

The graph of $y = c$, or $f(x) = c$, a horizontal line, is the graph of a function. Such a function is referred to as a **constant function**. The graph of $x = a$, a vertical line, is not the graph of a function.

R.4 Slope and Linear Functions

THEOREM 4

The graph of a function given by

$$y = mx \quad \text{or} \quad f(x) = mx$$

is the straight line through the origin (0, 0) and the point (1, m). The constant m is called the **slope** of the line.

R.4 Slope and Linear Functions

DEFINITION

The variable y **varies directly** as x if there is some positive constant m such that $y = mx$. We also say that y is **directly proportional** to x.

R.4 Slope and Linear Functions

DEFINITION:

A **linear function** is given by

$$y = mx + b \quad \text{or} \quad f(x) = mx + b$$

and has a graph that is the straight line parallel to the graph of $y = mx$ and crossing the x-axis at $(0, b)$. The point $(0, b)$ is called the ***y*-intercept**.

R.4 Slope and Linear Functions

DEFINITION:

$y = mx + b$ is called the **slope-intercept equation** of a line.

R.4 Slope and Linear Functions

DEFINITION:

$y - y_1 = m(x - x_1)$ is called the **point-slope equation** of a line.

R.4 Slope and Linear Functions

THEOREM 5

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \begin{array}{l}\text{slope of line containing} \\ \text{points } (x_1, y_1) \text{ and } (x_2, y_2)\end{array}$$

R.5 Nonlinear Functions and Models

OBJECTIVES

- Graph functions and solve applied problems.
- Manipulate radical expressions and rational exponents.
- Determine the domain of a rational function and graph certain rational functions.
- Find the equilibrium point given a supply function and a demand function.

R.5 Nonlinear Functions and Models

DEFINITION:

A **quadratic function** f is given by

$$f(x) = ax^2 + bx + c, \quad \text{where } a \neq 0.$$

R.5 Nonlinear Functions and Models

The graph of a **quadratic function** $f(x) = ax^2 + bx + c$ is called a **parabola**.

a) It is always a cup-shaped curve.

b) It opens upward if $a > 0$ or opens downward if $a < 0$.

c) It has a turning point, or **vertex**, at a point with first coordinate

$$x = -\frac{b}{2a}.$$

d) The vertical line $x = -b/(2a)$ serves as the line of symmetry.

R.5 Nonlinear Functions and Models

THEOREM 6: The Quadratic Formula

The solutions of any quadratic equation

$$ax^2 + bx + c = 0, \quad a \neq 0,$$

are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

R.5 Nonlinear Functions and Models

DEFINITION:

A **polynomial function** f is given by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x^1 + a_0.$$

where n is a nonnegative integer and $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers, called **coefficients**.

R.5 Nonlinear Functions and Models

DEFINITION:

Functions given by the quotient, or ratio, of two polynomials are called **rational functions**.

R.5 Nonlinear Functions and Models

DEFINITION:

y **varies inversely** as *x* if there is some positive number *k* such that $y = k/x$. We also say that *y* is **inversely proportional** to *x*.

R.5 Nonlinear Functions and Models

For any nonzero real number a and any integers n and m,

$$a^n \cdot a^m = a^{n+m};$$

$$\frac{a^n}{a^m} = a^{n-m};$$

$$\left(a^n\right)^m = a^{n \cdot m};$$

$$a^{-m} = \frac{1}{a^m}.$$

R.6

Mathematical Models and Curve Fitting

OBJECTIVE

- Use curve fitting to find a mathematical model for a set of data and use the model to make predictions.