EXERCISES 2.1

1. Determine whether each of the following sentences is a statement.

a) In 2003 George W. Bush was the president of the United States.

b) $x + 3$ is a positive integer.

c) Fifteen is an even number.

d) If Jennifer is late for the party, then her cousin Zachary will be quite angry.

e) What time is it?

f) As of June 30, 2003, Christine Marie Evert had won the French Open a record seven times.

2. Identify the primitive statements in Exercise 1.

3. Let p, q be primitive statements for which the implication $p \rightarrow q$ is false. Determine the truth values for each of the following.

a) $p \wedge q$ b) $\neg p \vee q$ c) $q \rightarrow p$ d) $\neg q \rightarrow \neg p$

4. Let p, q, r, s denote the following statements:

p: I finish writing my computer program before lunch.
q: I shall play tennis in the afternoon.
r: The sun is shining.
s: The humidity is low.

Write the following in symbolic form.

a) If the sun is shining, I shall play tennis this afternoon.

b) Finishing the writing of my computer program before lunch is necessary for my playing tennis this afternoon.

c) Low humidity and sunshine are sufficient for me to play tennis this afternoon.

5. Let p, q, r denote the following statements about a particular triangle ABC.

p: Triangle ABC is isosceles.
q: Triangle ABC is equilateral.
r: Triangle ABC is equiangular.

Translate each of the following into an English sentence.

a) $q \rightarrow p$ b) $\neg p \rightarrow \neg q$

c) $q \leftrightarrow r$ d) $p \wedge \neg q$

e) $r \rightarrow p$

6. Determine the truth value of each of the following implications.

a) If $3 + 4 = 12$, then $3 + 2 = 6$.

b) If $3 + 3 = 6$, then $3 + 4 = 9$.

c) If Thomas Jefferson was the third president of the United States, then $2 + 3 = 5$.

7. Rewrite each of the following statements as an implication in the **if-then** form.

a) Practicing her serve daily is a sufficient condition for Darci to have a good chance of winning the tennis tournament.

b) Fix my air conditioner or I won't pay the rent.

c) Mary will be allowed on Larry's motorcycle only if she wears her helmet.

8. Construct a truth table for each of the following compound statements, where p, q, r denote primitive statements.

a) $\neg(p \vee \neg q) \rightarrow \neg p$ b) $p \rightarrow (q \rightarrow r)$

c) $(p \rightarrow q) \rightarrow r$ d) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

e) $[p \wedge (p \rightarrow q)] \rightarrow q$ f) $(p \wedge q) \rightarrow p$

g) $q \leftrightarrow (\neg p \vee \neg q)$

h) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

9. Which of the compound statements in Exercise 8 are tautologies?

10. Verify that $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology.

11. a) How many rows are needed for the truth table of the compound statement $(p \vee \neg q) \leftrightarrow [(\neg r \wedge s) \rightarrow t]$, where p, q, r, s, and t are primitive statements?

b) Let $p_1, p_2, \ldots, p_n$ denote n primitive statements. Let p be a compound statement that contains at least one occurrence each of p_i, for $1 \leq i \leq n$—and p contains no other primitive statement. How many rows are needed to construct the truth table for p?

12. Determine all truth value assignments, if any, for the primitive statements p, q, r, s, t that make each of the following compound statements false.

a) $[(p \wedge q) \wedge r] \rightarrow (s \vee t)$

b) $[p \wedge (q \wedge r)] \rightarrow (s \veebar t)$

13. If statement q has the truth value 1, determine all truth value assignments for the primitive statements, p, r, and s for which the truth value of the statement

$$(q \rightarrow [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \rightarrow (\neg r \wedge q)]$$

is 1.

14. At the start of a program (written in pseudocode) the integer variable n is assigned the value 7. Determine the value of n after each of the following *successive* statements is encountered during the execution of this program. [Here the value of n following the execution of the statement in part (a) becomes the value of n for the statement in part (b), and so on, through the statement in part (d). For positive integers a, b, $\lfloor a/b \rfloor$ returns the integer part of the quotient—for example, $\lfloor 6/2 \rfloor = 3$, $\lfloor 7/2 \rfloor = 3$, $\lfloor 2/5 \rfloor = 0$, and $\lfloor 8/3 \rfloor = 2$.]

a) **if** $n > 5$ **then** $n := n + 2$