flows from T_1 to T_2 in network (a) exactly when it does so in network (b). But network (b) has only four switches, five fewer than network (a).

EXERCISES 2.2

1. Let p, q, r denote primitive statements.

a) Use truth tables to verify the following logical equivalences.

i) $p \rightarrow (q \wedge r) \Longleftrightarrow (p \rightarrow q) \wedge (p \rightarrow r)$
ii) $[(p \vee q) \rightarrow r] \Longleftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$
iii) $[p \rightarrow (q \vee r)] \Longleftrightarrow [\neg r \rightarrow (p \rightarrow q)]$

b) Use the substitution rules to show that

$$[p \rightarrow (q \vee r)] \Longleftrightarrow [(p \wedge \neg q) \rightarrow r].$$

2. Verify the first Absorption Law by means of a truth table.

3. Use the substitution rules to verify that each of the following is a tautology. (Here p, q, and r are primitive statements.)

a) $[p \vee (q \wedge r)] \vee \neg[p \vee (q \wedge r)]$

b) $[(p \vee q) \rightarrow r] \leftrightarrow [\neg r \rightarrow \neg(p \vee q)]$

4. For primitive statements p, q, r, and s, simplify the compound statement

$$[[[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]] \vee \neg q] \rightarrow s.$$

5. Negate and express each of the following statements in smooth English.

a) Kelsey will get a good education if she puts her studies before her interest in cheerleading.

b) Norma is doing her homework, and Karen is practicing her piano lessons.

c) If Harold passes his C++ course and finishes his data structures project, then he will graduate at the end of the semester.

6. Negate each of the following and simplify the resulting statement.

a) $p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)$

b) $(p \wedge q) \rightarrow r$

c) $p \rightarrow (\neg q \wedge r)$

d) $p \vee q \vee (\neg p \wedge \neg q \wedge r)$

7. a) If p, q are primitive statements, prove that

$$(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \Longleftrightarrow (p \wedge q).$$

b) Write the dual of the logical equivalence in part (a).

8. Write the dual for (a) $q \rightarrow p$, (b) $p \rightarrow (q \wedge r)$, (c) $p \leftrightarrow q$, and (d) $p \veebar q$, where p, q, and r are primitive statements.

9. Write the converse, inverse, and contrapositive of each of the following implications. For each implication, determine its truth value as well as the truth values of its corresponding converse, inverse, and contrapositive.

a) If $0 + 0 = 0$, then $1 + 1 = 1$.

b) If $-1 < 3$ and $3 + 7 = 10$, then $\sin\left(\frac{3\pi}{2}\right) = -1$.

10. Determine whether each of the following is true or false. Here p, q are arbitrary statements.

a) An equivalent way to express the converse of "p is sufficient for q" is "p is necessary for q."

b) An equivalent way to express the inverse of "p is necessary for q" is "$\neg q$ is sufficient for $\neg p$."

c) An equivalent way to express the contrapositive of "p is necessary for q" is "$\neg q$ is necessary for $\neg p$."

11. Let p, q, and r denote primitive statements. Find a form of the contrapositive of $p \rightarrow (q \rightarrow r)$ with (a) only one occurrence of the connective $\rightarrow$; (b) no occurrences of the connective $\rightarrow$.

12. Show that for primitive statements p, q,

$$p \veebar q \Longleftrightarrow [(p \wedge \neg q) \vee (\neg p \wedge q)] \Longleftrightarrow \neg(p \leftrightarrow q).$$

13. Verify that $[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \Longleftrightarrow [(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)]$, for primitive statements p, q, and r.

14. For primitive statements p, q,

a) verify that $p \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology.

b) verify that $(p \vee q) \rightarrow [q \rightarrow q]$ is a tautology by using the result from part (a) along with the substitution rules and the laws of logic.

c) is $(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$ a tautology?

15. Define the connective "Nand" or "Not ... and ..." by $(p \uparrow q) \Longleftrightarrow \neg(p \wedge q)$, for any statements p, q. Represent the following using only this connective.

a) $\neg p$ **b)** $p \vee q$ **c)** $p \wedge q$
d) $p \rightarrow q$ **e)** $p \leftrightarrow q$

16. The connective "Nor" or "Not ... or ..." is defined for any statements p, q by $(p \downarrow q) \Longleftrightarrow \neg(p \vee q)$. Represent the statements in parts (a) through (e) of Exercise 15, using only this connective.

17. For any statements p, q, prove that

a) $\neg(p \downarrow q) \Longleftrightarrow (\neg p \uparrow \neg q)$

b) $\neg(p \uparrow q) \Longleftrightarrow (\neg p \downarrow \neg q)$

18. Give the reasons for each step in the following simplifications of compound statements.

a)

	Reasons
$[(p \vee q) \wedge (p \vee \neg q)] \vee q$	
$\Longleftrightarrow [p \vee (q \wedge \neg q)] \vee q$	
$\Longleftrightarrow (p \vee F_0) \vee q$	
$\Longleftrightarrow p \vee q$	