

Continuing with the premise $r \rightarrow \neg s$, we find that because s has the truth value 1, the truth value of r must be 0. Hence r is false. But with $\neg p$ false and the premise $\neg p \vee r$ true, we also have r true. Therefore we find that $p \Rightarrow (\neg r \wedge r)$.

We have failed in our attempt to find a counterexample to the validity of the given argument. However, this failure has shown us that the given argument is valid—and the validity follows by using the method of Proof by Contradiction.

This introduction to the rules of inference has been far from exhaustive. Several of the books cited among the references listed near the end of this chapter offer additional material for the reader who wishes to pursue this topic further. In Section 2.5 we shall apply the ideas developed in this section to statements of a more mathematical nature. For we shall want to learn how to develop a proof for a theorem. And then in Chapter 4 another very important proof technique called *mathematical induction* will be added to our arsenal of weapons for proving mathematical theorems. First, however, the reader should carefully complete the exercises for this section.

EXERCISES 2.3

1. The following are three valid arguments. Establish the validity of each by means of a truth table. In each case, determine which rows of the table are crucial for assessing the validity of the argument and which rows can be ignored.

- a) $[p \wedge (p \rightarrow q) \wedge r] \rightarrow [(p \vee q) \rightarrow r]$
- b) $[[p \wedge q] \rightarrow r] \wedge \neg q \wedge (p \rightarrow \neg r) \rightarrow (\neg p \vee \neg q)$
- c) $[[p \vee (q \vee r)] \wedge \neg q] \rightarrow (p \vee r)$

2. Use truth tables to verify that each of the following is a logical implication.

- a) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
- b) $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$
- c) $[(p \vee q) \wedge \neg p] \rightarrow q$
- d) $[(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(p \vee q) \rightarrow r]$

3. Verify that each of the following is a logical implication by showing that it is impossible for the conclusion to have the truth value 0 while the hypothesis has the truth value 1.

- a) $(p \wedge q) \rightarrow p$
- b) $p \rightarrow (p \vee q)$
- c) $[(p \vee q) \wedge \neg p] \rightarrow q$
- d) $[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (q \vee s)$
- e) $[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\neg q \vee \neg s)] \rightarrow (\neg p \vee \neg r)$

4. For each of the following pairs of statements, use Modus Ponens or Modus Tollens to fill in the blank line so that a valid argument is presented.

- a) If Janice has trouble starting her car, then her daughter Angela will check Janice's spark plugs.
Janice had trouble starting her car.

\therefore _____

- b) If Brady solved the first problem correctly, then the answer he obtained is 137.

Brady's answer to the first problem is not 137.

\therefore _____

- c) If this is a **repeat-until** loop, then the body of this loop is executed at least once.

\therefore The body of the loop is executed at least once.

- d) If Tim plays basketball in the afternoon, then he will not watch television in the evening.

\therefore Tim didn't play basketball in the afternoon.

5. Consider each of the following arguments. If the argument is valid, identify the rule of inference that establishes its validity. If not, indicate whether the error is due to an attempt to argue by the converse or by the inverse.

- a) Andrea can program in C++, and she can program in Java.

Therefore Andrea can program in C++.

- b) A sufficient condition for Bubbles to win the golf tournament is that her opponent Meg not sink a birdie on the last hole.

Bubbles won the golf tournament.

Therefore Bubbles' opponent Meg did not sink a birdie on the last hole.

- c) If Ron's computer program is correct, then he'll be able to complete his computer science assignment in at most two hours.

It takes Ron over two hours to complete his computer science assignment.

Therefore Ron's computer program is not correct.

- d) Eileen's car keys are in her purse, or they are on the kitchen table.