

Eileen's car keys are not on the kitchen table.
Therefore Eileen's car keys are in her purse.

e) If interest rates fall, then the stock market will rise.
Interest rates are not falling.
Therefore the stock market will not rise.

6. For primitive statements p , q , and r , let P denote the statement

$$[p \wedge (q \wedge r)] \vee \neg[p \vee (q \wedge r)],$$

while P_1 denotes the statement

$$[p \wedge (q \vee r)] \vee \neg[p \vee (q \vee r)].$$

a) Use the rules of inference to show that

$$q \wedge r \Rightarrow q \vee r.$$

b) Is it true that $P \Rightarrow P_1$?

7. Give the reason(s) for each step needed to show that the following argument is valid.

$$[p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \neg q)] \rightarrow (s \vee t)$$

Steps	Reasons
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- | | |
|---------------------------|--|
| 1) p | |
| 2) $p \rightarrow q$ | |
| 3) q | |
| 4) $r \rightarrow \neg q$ | |
| 5) $q \rightarrow \neg r$ | |
| 6) $\neg r$ | |
| 7) $s \vee r$ | |
| 8) s | |
| 9) $\therefore s \vee t$ | |

8. Give the reasons for the steps verifying the following argument.

$$\begin{array}{l} (\neg p \vee q) \rightarrow r \\ r \rightarrow (s \vee t) \\ \neg s \wedge \neg u \\ \neg u \rightarrow \neg t \\ \hline \therefore p \end{array}$$

Steps	Reasons
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| 1) $\neg s \wedge \neg u$ | |
| 2) $\neg u$ | |
| 3) $\neg u \rightarrow \neg t$ | |
| 4) $\neg t$ | |
| 5) $\neg s$ | |
| 6) $\neg s \wedge \neg t$ | |
| 7) $r \rightarrow (s \vee t)$ | |
| 8) $\neg(s \vee t) \rightarrow \neg r$ | |
| 9) $(\neg s \wedge \neg t) \rightarrow \neg r$ | |
| 10) $\neg r$ | |
| 11) $(\neg p \vee q) \rightarrow r$ | |
| 12) $\neg r \rightarrow \neg(\neg p \vee q)$ | |
| 13) $\neg r \rightarrow (p \wedge \neg q)$ | |
| 14) $p \wedge \neg q$ | |
| 15) $\therefore p$ | |

9. a) Give the reasons for the steps given to validate the argument

$$[(p \rightarrow q) \wedge (\neg r \vee s) \wedge (p \vee r)] \rightarrow (\neg q \rightarrow s).$$

Steps	Reasons
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|---------------------------------------|--|
| 1) $\neg(\neg q \rightarrow s)$ | |
| 2) $\neg q \wedge \neg s$ | |
| 3) $\neg s$ | |
| 4) $\neg r \vee s$ | |
| 5) $\neg r$ | |
| 6) $p \rightarrow q$ | |
| 7) $\neg q$ | |
| 8) $\neg p$ | |
| 9) $p \vee r$ | |
| 10) r | |
| 11) $\neg r \wedge r$ | |
| 12) $\therefore \neg q \rightarrow s$ | |

b) Give a direct proof for the result in part (a).

c) Give a direct proof for the result in Example 2.32.

10. Establish the validity of the following arguments.

a) $[(p \wedge \neg q) \wedge r] \rightarrow [(p \wedge r) \vee q]$

b) $[p \wedge (p \rightarrow q) \wedge (\neg q \vee r)] \rightarrow r$

c) $p \rightarrow q$	d) $p \rightarrow q$
$\neg q$	$r \rightarrow \neg q$
$\neg r$	r
$\hline \therefore \neg(p \vee r)$	$\hline \therefore \neg p$

e) $p \rightarrow (q \rightarrow r)$	f) $p \wedge q$
$\neg q \rightarrow \neg p$	$p \rightarrow (r \wedge q)$
p	$r \rightarrow (s \vee t)$
$\hline \therefore r$	$\neg s$
	$\hline \therefore t$

g) $p \rightarrow (q \rightarrow r)$	h) $p \vee q$
$p \vee s$	$\neg p \vee r$
$t \rightarrow q$	$\neg r$
$\neg s$	$\hline \therefore q$
$\hline \therefore \neg r \rightarrow \neg t$	

11. Show that each of the following arguments is invalid by providing a counterexample—that is, an assignment of truth values for the given primitive statements p , q , r , and s such that all premises are true (have the truth value 1) while the conclusion is false (has the truth value 0).

a) $[(p \wedge \neg q) \wedge [p \rightarrow (q \rightarrow r)]] \rightarrow \neg r$

b) $[(p \wedge q) \rightarrow r] \wedge (\neg q \vee r) \rightarrow p$

c) $p \leftrightarrow q$	d) p
$q \rightarrow r$	$p \rightarrow r$
$r \vee \neg s$	$p \rightarrow (q \vee \neg r)$
$\neg s \rightarrow q$	$\neg q \vee \neg s$
$\hline \therefore s$	$\hline \therefore s$