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Department of Mathematical Sciences

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Midterm for MA1101, Basic calculus I

English
Tuesday 11 October 2011
Time: 08.15 – 09.45
Permitted aids: Calculator (HP30S or Citizen SR-270X)

Show all work and explain your reasoning. Good luck!

Problem 1

a) $f(x) = \sqrt{2x^2 + 1}$. Determine $f'(2)$.

b) $\int (\sin x + x^2)\, dx =$

c) $\int 2x \cos(x^2)\, dx =$

Problem 2

a) Prove that the function

$$f(x) = 3x^3 + x - 1$$

is increasing for all values of x.

b) Prove that the function f given in **a)** has precisely one zero on the interval $(0, 1)$.

Problem 3

Use the formal definition of limit (with ϵ and δ) to determine

$$\lim_{x \to 2} \frac{x-2}{x^2+1}$$

Show that in this case one may take $\delta = \epsilon$.

Problem 4

The following formula is given:

$$\cos 2x = \cos^2 x - \sin^2 x$$

(A proof of this formula is not required.)
Use this formula to prove that:

$$\sin 2x = 2 \sin x \cos x$$

Problem 5

a) Write down the Mean-Value Theorem. (Without proof.)

b) Suppose that the function f is defined and differentiable on the open interval (a, b) and that

$$f'(x) < 0 \quad \text{for all} \quad x \in (a, b)$$

Use the Mean-Value Theorem to prove that f is then a decreasing function on (a, b), i.e. if $x_1 < x_2$ then $f(x_1) > f(x_2)$.

c) Determine if the following statement is true: If f is differentiable on an open interval, and decreasing on this interval, then $f'(x) < 0$ for all x in this interval. Give a proof or a counterexample.

Problem 6

The parabola

$$y = x^2$$

has two tangent lines that pass through the point $(1, -8)$. Determine the equation of these tangent lines.