

[Good to reset everything, if needed:

```
> restart;
```

[Load the package VectorCalculus, which is very good for Math 2.

```
> with(VectorCalculus);
```

```
[&x, `*`, `+`, `-', `.` , <, >, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, Compatibility, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series ]
```

(1)

[Load the package plots, which is always very good.

```
> with(plots);
```

```
[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]
```

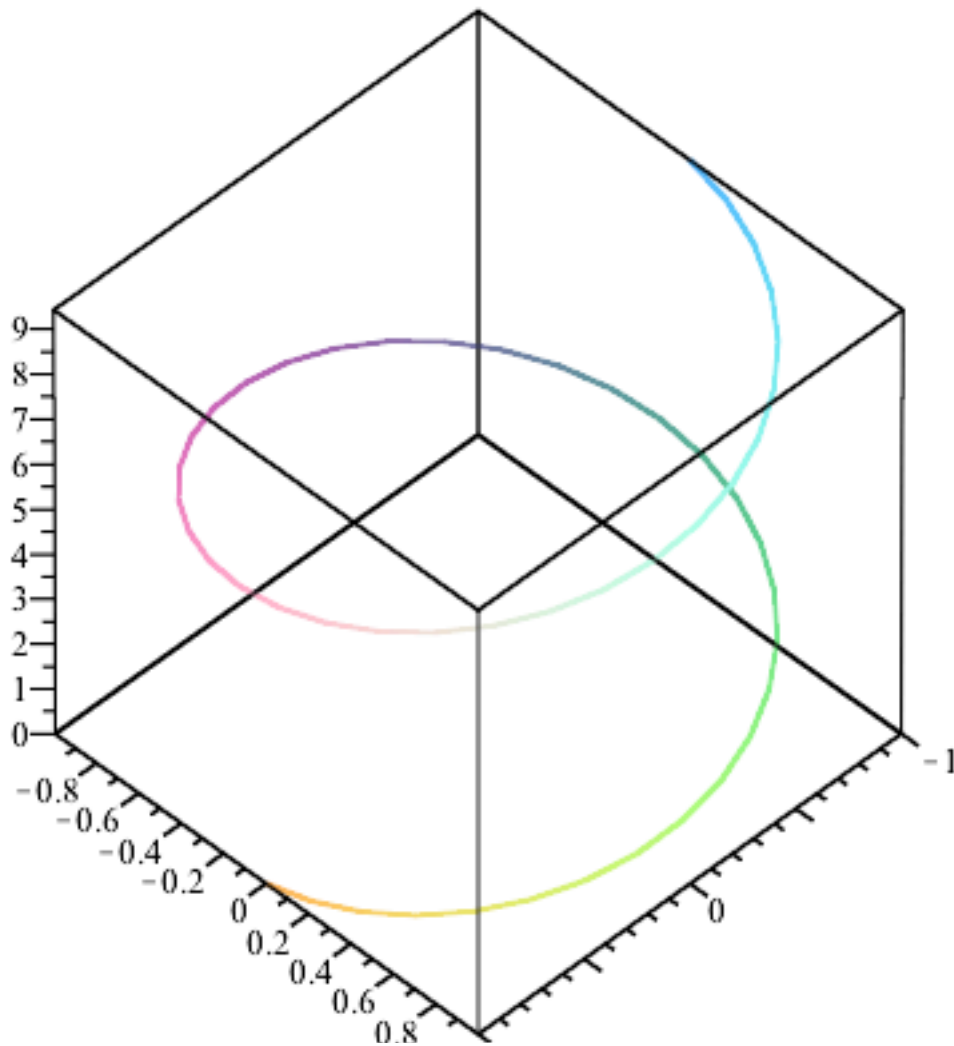
(2)

[Define a function/expression in t (a helix curve).

```
> f(t) := <cos(t), sin(t), t>;
```

[Plot the corresponding curve in space, on some interval, with boxed axes.

```
> spacecurve(f(t), t=0..3*Pi, axes=boxed);
```



Calculate (and simplify) some obvious entities.

> **TangentVector(f(t), normalized);**

$$1$$

(3)

> **PrincipalNormal(f(t), normalized);**

$$h(t) := \begin{bmatrix} -\cos(t) \\ -\sin(t) \\ 0 \end{bmatrix}$$

(4)

> **Curvature(f(t));**

$$\frac{1}{4} \sqrt{2 \cos(t)^2 + 2 \sin(t)^2} \sqrt{2}$$

(5)

> **simplify(%);**

$$\frac{1}{2}$$

(6)

> **Binormal(f(t), normalized);**

$$j(t) := \begin{bmatrix} \frac{1}{2} \sqrt{2} \sin(t) \\ -\frac{1}{2} \sqrt{2} \cos(t) \\ \frac{1}{2} \sqrt{2} \end{bmatrix} \quad (7)$$

> Torsion(f(t));

$$\frac{1}{2} \frac{\sqrt{2} \sin(t)^2}{\sqrt{2 \cos(t)^2 + 2 \sin(t)^2} \sqrt{\left(\frac{1}{4} + \frac{1}{4} \sin(t)^2 + \frac{1}{4} \cos(t)^2\right) (1 + \sin(t)^2 + \cos(t)^2)} + \frac{1}{2} (\sqrt{2} \cos(t)^2) / \left(\sqrt{2 \cos(t)^2 + 2 \sin(t)^2} \sqrt{\left(\frac{1}{4} + \frac{1}{4} \sin(t)^2 + \frac{1}{4} \cos(t)^2\right) (1 + \sin(t)^2 + \cos(t)^2)} \right) \quad (8)$$

> simplify(%);

$$\frac{1}{2} \quad (9)$$

Below, a different way of defining the same curve:

> helix:=PositionVector([cos(t),sin(t),t]);

$$helix := \begin{bmatrix} \cos(t) \\ \sin(t) \\ t \end{bmatrix} \quad (10)$$

Plotting the curve with the additional choices 'tangent', 'normal' and 'binormal' shows the geometrical meaning of the TNB-frame along a curve in three-dimensional space.

> PlotPositionVector(helix, t=0..Pi, points=[Pi/2], tangent,normal,binormal);

