NTNU
Norwegian University of
Science and Technology

Department of Mathematical Sciences

Examination paper for **MA1103 Flerdimensjonal analyse**

Academic contact during examination:

Phone:

Examination date: August 2017

Examination time (from–to): 09:00–13:00

Permitted examination support material: D: No printed or hand-written support material is allowed. A specific basic calculator is allowed.

Other information:
You must give reasons for all answers. You can answer in Norwegian if you prefer to. A list of formulas is attached.

Language: English

Number of pages: 2

Number of pages enclosed: 1

Checked by:

Informasjon om trykking av eksamensoppgave	
Originalen er:	
1-sidig ☐	**2-sidig** ☒
sort/hvit ☒	**farger** ☐
skal ha flervalgskjema ☐	

Date Signature

Problem 1 Determine whether

$$f(x,y) := \begin{cases} \frac{x^3 - x^2y + 4xy^2}{3x^2y + y^3}, & \text{if} \quad (x,y) \neq (0,0) \\ 0, & \text{if} \quad (x,y) = (0,0) \end{cases}$$

is continuous at $(0,0)$ or not.

Problem 2 The path of a fly is the curve C parameterized by

$$\mathbf{r}(t) = (e^t \cos(t), e^t \sin(t), 1), \qquad t \geq 0.$$

a) Compute the velocity vector, the speed, and the acceleration of the fly at time $t = 0$.

b) The curvature κ of C at time t is defined as

$$\kappa(t) := \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}.$$

Compute the curvature of the curve C.

Problem 3 Let $f(x,y) = (x^2 + y^2)e^x$

a) Find and classify all critical points.

b) Find the tangent plane of the graph $z = f(x,y)$ at the point $(0,1,1)$.

Problem 4 Let D be the region in $\mathbb{R}^2$ determined by the inequalities $x^2 + y^2 \leq 1$ and $y \geq x$. Sketch the region D and evaluate the following integral:

$$\iint_D 2\cos(x^2 + y^2)d(x,y).$$

Problem 5 Compute the area of the surface S given by $z = 2\sqrt{x}$, where $x \in [0,1]$ and $y \in [0, \sqrt{x}]$.

Problem 6 Let $\mathbf{F}(x, y, z) = (zy^2e^{xz}, 2ye^{xz}, xy^2e^{xz})$.

i) Show that $\mathbf{F}$ is a conservative field and find a potential function f such that $\mathbf{F} = \nabla f$.

ii) Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where C is a curve parameterized by

$$\mathbf{r}(t) = (\cos(2t), \sin(t), t(\pi - 2t)), \qquad t \in \left[0, \frac{\pi}{2}\right].$$

Problem 7 Let R be the region in $\mathbb{R}^3$ enclosed by $z = 1 - e^{1-x^2-y^2}$ and $z = 0$.

a) Sketch the region and compute the volume of R.

b) Let the vector field $\mathbf{F}$ be given by

$$\mathbf{F}(x, y, z) = (2x + e^{yz}, \sin(y), x - \cos(y)z).$$

Evaluate

$$\iint_{\partial R} \mathbf{F} \cdot \mathbf{n}\, d\mathbf{S},$$

where ∂R denotes the boundary of R.

List of formulas for MA1103 Vector Calculus 2017

Taylor's formula.

First-order Taylor approximation at $\boldsymbol{x}^0 \in \mathbb{R}^n$.

$$T_1(\mathbf{x}) = f(\mathbf{x}^0) + \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(\mathbf{x}^0)(x_i - x_i^0).$$

Second-order Taylor approximation at $\boldsymbol{x}^0 \in \mathbb{R}^n$.

$$T_2(\mathbf{x}) = f(\mathbf{x}^0) + \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(\mathbf{x}^0)(x_i - x_i^0) + \frac{1}{2}\sum_{i,j=1}^{n} \frac{\partial^2 f}{\partial x_i \partial x_j}(\mathbf{x}^0)(x_i - x_i^0)(x_j - x_j^0).$$

Change of variables formula.

Change of variables formula for polar coordinates.

$$\iint_D f(x,y)\, d(x,y) = \iint_{D^*} f(r\cos\theta, r\sin\theta) r\, d(r,\theta).$$

Change of variables formula for cylindrical coordinates.

$$\iiint_W f(x,y,z)\, d(x,y,z) = \iiint_{W^*} f(r\cos\theta, r\sin\theta, z) r\, d(r,\theta,z).$$

Change of variables formula for spherical coordinates.

$$\iiint_W f(x,y,z)\, d(x,y,z) = \iiint_{W^*} f(r\sin\phi\cos\theta, r\sin\phi\sin\theta, r\cos\phi) r^2 \sin\phi\, d(r,\theta,\phi).$$

Integrals over curves. Let C be a curve parameterized by $\mathbf{r} : [a,b] \to \mathbb{R}^n$.

Scalar fields. $f : C \to \mathbb{R}$

$$\int_C f\, ds = \int_a^b f(\mathbf{r}(t)) \|\mathbf{r}'(t)\|\, dt.$$

Vector fields. $\mathbf{F} : C \subset \mathbb{R}^n \to \mathbb{R}^n$

$$\int_C \mathbf{F} \cdot\, d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\, dt.$$

Integrals over surfaces. Let T be a surface parameterized by $\mathbf{r} : A \subset \mathbb{R}^2 \to \mathbb{R}^3$.

Scalar fields. $f : T \to \mathbb{R}$

$$\iint_T f\, dS = \iint_A f(\mathbf{r}(u,v)) \left\| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right\| d(u,v).$$

Vector fields. $\mathbf{F} : T \subset \mathbb{R}^3 \to \mathbb{R}^3$

$$\iint_T \mathbf{F} \cdot \mathbf{n}\, d\mathbf{S} = \iint_A \mathbf{F}(\mathbf{r}(u,v)) \cdot \left(\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right) d(u,v).$$

Green's Theorem.

$$\iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) d(x,y) = \int_{\partial D} \mathbf{F} \cdot d\mathbf{r}.$$

Stokes' Theorem.

$$\iint_S (\operatorname{curl} \mathbf{F}) \cdot \mathbf{n}\, d\mathbf{S} = \int_{\partial S} \mathbf{F} \cdot d\mathbf{r}.$$

Divergence Theorem.

$$\iiint_V \operatorname{div} \mathbf{F}\, d(x,y,z) = \iint_{\partial V} \mathbf{F} \cdot \mathbf{n}\, d\mathbf{S}.$$