BEVIS FOR TEOREM 3.4.1 (d): (Oppg. 26, s. 155)

La $\mathbf{u} = (u_1, u_2, u_3)$, $\mathbf{v} = (v_1, v_2, v_3)$, $\mathbf{w} = (w_1, w_2, w_3)$

Vi har:

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = (v_2 w_3 - v_3 w_2)\mathbf{i} + (v_3 w_1 - v_1 w_3)\mathbf{j} + (v_1 w_2 - v_2 w_1)\mathbf{k}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ (v_2 w_3 - v_3 w_2) & (v_3 w_1 - v_1 w_3) & (v_1 w_2 - v_2 w_1) \end{vmatrix}$$

$$\begin{aligned} &= [u_2(v_1 w_2 - v_2 w_1) - u_3(v_3 w_1 - v_1 w_3)]\mathbf{i} \\ &+ [u_3(v_2 w_3 - v_3 w_2) - u_1(v_1 w_2 - v_2 w_1)]\mathbf{j} \\ &+ [u_1(v_3 w_1 - v_1 w_3) - u_2(v_2 w_3 - v_3 w_2)]\mathbf{k} \end{aligned}$$

$$\begin{aligned} &= [(u_2 w_2 + u_3 w_3)v_1 + (-u_2 v_2 - u_3 v_3)w_1]\mathbf{i} \\ &+ [(u_3 w_3 + u_1 w_1)v_2 + (-u_3 v_3 - u_1 v_1)w_2]\mathbf{j} \\ &+ [(u_1 w_1 + u_2 w_2)v_3 + (-u_1 v_1 - u_2 v_2)w_3] \end{aligned}$$

Legger til og trekker fra samme ledd i hver parantes

$$\begin{aligned} &= [(u_1 w_1 + u_2 w_2 + u_3 w_3)v_1 - (u_1 v_1 + u_2 v_2 + u_3 v_3)w_1]\mathbf{i} \\ &+ [(u_1 w_1 + u_2 w_2 + u_3 w_3)v_2 - (u_1 v_1 + u_2 v_2 + u_3 v_3)w_2]\mathbf{j} \\ &+ [(u_1 w_1 + u_2 w_2 + u_3 w_3)v_3 - (u_1 v_1 + u_2 v_2 + u_3 v_3)w_3]\mathbf{k} \\ &= (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w} \end{aligned}$$

e) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = -[\mathbf{w} \times (\mathbf{u} \times \mathbf{v})]$ (Oppg. 27, s. 155)

$$\begin{aligned} &= -[(\mathbf{w} \cdot \mathbf{v})\mathbf{u} - (\mathbf{w} \cdot \mathbf{u})\mathbf{v}] \\ &= (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{v} \cdot \mathbf{w})\mathbf{u} \end{aligned}$$