Fasit - V05 OBS: Ikke komplett løsning!

1a. $A \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ $x = 1$, $y = 2$, $z = 3$

b. $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & t+1 \\ 3 & 6 & (3t^2 - \frac{15}{2}) & 0 \end{array}\right] \sim \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17+t \\ 0 & 0 & 6(t^2-1) & -3(t+1) \end{bmatrix}$

$t = -1$: ∞ mange løsn.

$t = 1$: 0 løsn.

$t \neq \pm 1$: 1 løsning.

2a. $z = a + bi$ $w = c + di$ $(a, b, c, d \in \mathbb{R})$

$\det A = z\bar{z} + w\bar{w} = |z|^2 + |w|^2 > 0$ for $z \neq 0$ ~~...~~ eller $w \neq 0$ (eller begge)

b. $1 - i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)\right)$

$w_1 = \sqrt[6]{2}\left(\dots -\frac{\pi}{12} \dots \right)$

$w_2 = \dots \left(\dots \frac{7\pi}{12} \dots \right)$

$w_3 = \dots \left(\dots \frac{15\pi}{12} \dots \right)$

3a. $\lambda_1 = 2 \rightsquigarrow \underline{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2 = -2 \longrightarrow v_2 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}$ $P^{-1} = \frac{1}{-4}\begin{bmatrix} -3 & -1 \\ -1 & 1 \end{bmatrix}$

$\rightsquigarrow D = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

b.) $A^n = PD^nP^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 2^n & 0 \\ 0 & (-2)^n \end{bmatrix}\begin{bmatrix} 3/4 & 1/4 \\ 1/4 & -1/4 \end{bmatrix} =$ (regn ut)

c) • $\det(A) = -4 \neq 0 \Rightarrow A$ inverterbar

• La A være en inverterbar $n \times n$-matrise. La B og C være inverser, dvs $BA = AB = I = CA = AC$.

Da er $\underline{B = B \cdot I} = B \cdot AC = BA \cdot C = I \cdot C = \underline{\underline{C}}$

4. $[T_1] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\qquad [T_2] = \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$[T_1 \circ T_2] = [T_1]\cdot[T_2] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$$

$$[T_2 \circ T_1] = [T_2]\circ[T_1] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$\Rightarrow T_1 \circ T_2 \neq T_2 \circ T_1$

5. $\overrightarrow{P_1P_2} = (3,-1,-1)$ $\qquad \overrightarrow{P_1P_3} = (2,-1,-5)$

$$\underline{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} i & j & k \\ 3 & -1 & -1 \\ 2 & -1 & -5 \end{vmatrix} = (4, 13, -1)$$

$\underline{n} \cdot (x-1, y-1, z-0) = 0$

$(4, 13, -1)\cdot(x-1, y-1, z) = 0$

$4x - 4 + 13y - 13 - z = 0$

$\underline{\underline{4x + 13y - z - 17 = 0}}$