# MA2501 Numerical methods

Spring 2010

**Problem set 1**

The first two exercises contain some MATLAB-code. Use `help` or `helpdesk` for an explanation of the different elements in the code.

Remember that there is often more than one way of doing things in MATLAB.

### Exercise 1

Given the equation $f(x) = e^x + x^2 - x - 4 = 0$.

**a)** Show (theoretically) that $f$ has one unique root, $r$, in the interval $[1, 2]$.

Plot $f(x), x \in [1, 2]$, in MATLAB and find an approximation to $r$ by inspecting the plot. This can for example be done by writing

```
f = inline('exp(x)+x^2-x-4');
fplot(f,[1,2])
grid on
```

Find an approximation to $r$ by using Newton's method. Use $x_0 = 1.5$ as a starting value. (You may also use a calculator instead of MATLAB.)

**b)** The equation $f(x) = 0$ can be written on the form $x = g(x)$ with for instance

$$\begin{aligned} &i) && g(x) = \ln(4 + x - x^2) \\ &ii) && g(x) = \sqrt{-e^x + x + 4} \\ &iii) && g(x) = e^x + x^2 - 4 \end{aligned}$$

From each of these we can create a fixed-point iteration scheme $x_{n+1} = g(x_n)$. The MATLAB code may look like

```
g = inline('log(4+x-x^2)');
x = 1.5           % Startverdi
Nit = 10          % Antall iterasjoner
for n=1:Nit
  x = g(x)
end
```

Test the three iteration schemes numerically, and see if they converge or not. Use $x_0 = 1.5$, but you may also experiment with other starting values. Use Theorem 1 in the note about fixed-point iterations to verify the numerical results.

**Hint:** If necessary, choose another interval $[a, b]$ which contains $r$. You can also use MATLAB to plot $g(x)$ and $g'(x)$. For the converging scheme(s), estimate the maximum number of iterations necessary to approximate $r$ with accuracy $10^{-6}$. Compare with the numerical results. (The answer depends on the chosen interval $[a, b]$.)

**Exercise 2**

Let $p$ be a positive number. What is the value of this expression:

$$x = \sqrt{p + \sqrt{p + \sqrt{p + \cdots}}}$$

**Hint:** Notice that this may be written as a fixed-point iteration.

**Exercise 3**

Exercise 3.2.33, p.103, and 3.3.12, p.122 in the book.