MA2501 Numerical methods

Spring 2010

Problem set 3

Exercise 1

The system of equations $A\mathbf{x} = \mathbf{b}$ has $\mathbf{x}$ as the correct solution and $\tilde{\mathbf{x}}$ as an approximated solution.

a) See page 254 (chapter 7.1) and page 321 (chapter 8.2) in C &K. Show that

$$\frac{\|\mathbf{e}\|}{\|\mathbf{x}\|} \leq \kappa(A)\frac{\|\mathbf{r}\|}{\|\mathbf{b}\|}$$

where $\mathbf{e} = \mathbf{x} - \tilde{\mathbf{x}}$ og $\mathbf{r} = A\tilde{\mathbf{x}} - \mathbf{b}$.

b) Chapter 7.1, problem 5. Also compute the condition number to the matrix, and show that the inequality from **a)** is satisfied.

c) (MATLAB) The purpose of this exercise is to show that roundoff-error may be a significant problem for the solution of a system of equations if the coeffisientmatrix is badly conditioned.

The Hilbert-matrix is an $n \times n$ matrix with elements $a_{ij} = 1/(i + j - 1)$. Let A be an $n \times n$ Hilbert-matrix, $\mathbf{x}$ a vector of length n which you choose , and let $\mathbf{b} = A\mathbf{x}$. You now have a system of equations where you know the exact solution. Solve this problem in MATLAB. This will give you an *approximated* solution $\tilde{\mathbf{x}}$. How big is the error in the solution and the residual-error measured in the max-norm? Find also the condition-number to the matrix. Try this for $n = 5$, $n = 10$ and $n = 15$.

Repeat the experiment with a rando matrix.

Some useful MATLAB-commands:
`hilb(n)`: Creates an $n \times n$ Hilbert-matrix.
`norm(x,inf)`: Calculates the max-norm of a vector.
`cond(A,inf)`: Calculates the condition number of A by using the max-norm.

Exercise 2

Chapter 8.2, problems 3-9. (The answer in the back of the book for problem 9 is wrong)

Exercise 3

Given the system of equations on page 245 (the front page of Chapter 7) in C&K. If you try to solve this with Jacobi, Gauss-Seidel or SOR ($0 < \omega < 2$), will the iterations converge? Why/why not?

Exercise 4

Chapter 8.2, Computer problems 3 and 4.