MA2501 Numerical methods

Spring 2010

Problem set 4

Exercise 1

a) Use the Matlab-function `potens.m` to find the largest eigenvalue and the corresponding eigenvector for the matrix:

$$\begin{bmatrix} -2 & -2 & 3 \\ -10 & -1 & 6 \\ 10 & -2 & -9 \end{bmatrix}.$$

Use $\mathbf{x}^{(0)} = [1,0,0]^T$ as a startingvalue. Check the answer with Matlabs `eig`.

b) Repeat the experiment in **a)** with the matrix

$$\begin{bmatrix} 5 & 1 & -1 \\ 1 & 11 & 7 \\ -1 & 7 & 11 \end{bmatrix}$$

and with $\mathbf{x}^{(0)} = [1,1,1]^T$ as a startingvalue. This will probably not work. Why, and try to correct this. When everything is working, repeat with starting value $\mathbf{x}^{(0)} = [1,0,0]^T$. What is the answer after about 20 iterations, and what is the answer after 50? Explain.

c) Modify `potens.m` to use the shifted inverse power method to find eigenvalues. Use it to solve Computer Problem 8.4.4, p.369. Check the answers.

Exercise 2

a) Find the Lagrange form of the interpolation polynomial of the lowest possible degree that interpolates the table

x	0	2	3	4
y	7	11	28	63

b) Verify that the polynomials

$$p(x) = 5x^3 - 27x^2 + 45x - 21$$
$$q(x) = x^4 - 5x^3 + 8x^2 - 5x + 3$$

both interpolate the points in the table

x	1	2	3	4
y	2	1	6	47

Why doesn't this contradict the uniqueness theorem?

c) Solve Exercise 4.1.27 p.149 in the book.

Exercise 3

Exercise 4.2.13 p.162 in the book.

Exercise 4

a) Show that the functions $T_n(x)$, defined on $[-1, 1]$

$$T_n(x) = \cos(n \cdot \arccos(x)), \qquad \text{for } n = 0, 1, 2, \ldots,$$

are polynomials of degree n which satisfy the recursion formula

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_{n+1} = 2xT_n(x) - T_{n-1}(x).$$

These are the *Chebyshev polynomials*.

b) Plot the function

$$w_n(t) = \prod_{i=0}^{n} (t - x_i)$$

for $t \in [-1, 1]$ for each of the three cases

1. $x_i = \cos((2i + 1)\pi/(2(n + 1)))$ - (Chebyshev nodes, zeros of T_{n+1})
2. $x_i = \cos(i\pi/n)$
3. $x_i = -1 + 2i/n$

where $i = 0, 1, \ldots, n$. You may use the Matlab-function `w.m` on the web-page. For instance, for $n = 3$ you can plot $w_n(t)$ based on the Chebyshev-nodes by typing

```
>> n=3
>> i=0:n
>> x=cos((2*i+1)*pi/(2*(n+1)))
>> [t,y] = w(x, [-1,1])
>> plot(t,y,'b-','LineWidth',2)
>> grid on
```