MA2501 Numerical methods

Spring 2010

Problem set 6

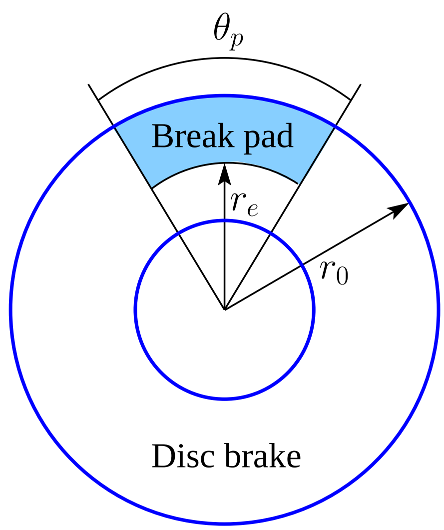
Exercise 1

In order to simulate thermal properties of a disc brake we need a numerical approximation of the average temperature over the break pad. This is given by

$$T = \frac{\int_{r_e}^{r_0} T(r) r \theta_p \, \mathrm{d}r}{\int_{r_e}^{r_0} r \theta_p \, \mathrm{d}r}$$

where $T(r)$ is the temperature at a position on the break pad. Here $r_e = 9.38cm$, $r_0 = 14.58cm$ and $\theta_p = 0.7051$ (radians). $T(r)$ for a few values r is given in the following Table (these may for example be the result of a numerical solution of the heat-equation):

r(cm)	$T(r)$($^\circ C$)
9.38	338
9.90	423
10.42	474
10.94	506
11.46	557
11.98	573
12.50	601
13.02	622
13.54	651
14.06	661
14.58	671



Use these values to find an approximation to the average temperature T (You may for instance use the function `trapz` in Matlab).

Exercise 2

Given $f(x) = e^{-x^2}$ in the points $x = 0.0, 0.2, 0.4, 0.6$ and 0.8.

a) Find an approximation to the integral

$$\int_0^{0.8} f(x)\mathrm{d}x$$

by using

1. Trapezoidal rule
2. Simpsons rule
3. Romberg algorithm

b) If we use Romberg-integration and all the given values the answer will have an error of approximately $2 \cdot 10^{-6}$. How many intervals does the trapezoidal rule need (using a uniform spacing) to achieve this error?

Exercise 3

Write a Matlab-program which performs Romberg-integration (you may for instance start with the algorithm on p.206 in the book). Test the program on the integrals in Computer Problems 5.3.2 and 5.3.3 (p.214).