**MA2501** Numeriske Metoder
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# Training Assignment 2

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This assignment has 5 tasks.

**Exercise 1**. Write down Newton's method for find solutions to the following equation system:

$$x^2 + (y-8)^2 = 8$$
$$x + .15x^2 + xy = 1$$

**Exercise 2**. What is the following code doing?

```python
x = 10.
for i in range(10):
    y = x**2
```

What is the value of **y** after the code is executed? What do you think of the usefulness of that code?

**Exercise 3**. Write down the simplified Newton's algorithm (i.e., when the derivative of the function is only computed at the first point $x_0$), in one dimension (i.e., we are looking for a root of a function from $\mathbf{R}$ to $\mathbf{R}$). Express it as a fixed point method. What is the condition for this fixed point method to converge? Will this method always converge if the starting point is sufficiently close to a root of $f$?

**Exercise 4**. Consider the floating point system with 3 significant decimal digits and 2 decimal exponents (i.e., the possible exponents range from -50 to 49), and without tricks, so we cannot represent the zero.

**4.a)** Prove that if the first digit is not allowed to be zero, two different set of digits lead to two different numbers.

**4.b)** Assuming that the first significant digit is not zero, what is

- The smallest possible positive, non-zero number?

- The smallest number strictly greater than one?

- The value of the *machine epsilon*?

- The biggest possible number?

**Exercise 5**. A $2 \times 2$ matrix $A$ and a vector $b \in \mathbf{R}^2$ are fixed. Consider the fixed point problem

$$Ax + b = x.$$

**5.a)** What is the condition for this problem to have a solution? Find an example of $A$ and $b$ such that there is no solution.

**5.b)** What is the condition for the fixed point iteration

$$x_{n+1} = Ax_n + b$$

to converge to a given fixed point $\bar{x}$?