**Norwegian University of Science and Technology**
**Department of Mathematical Sciences**

**MA2501** Numeriske Metoder
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# Training Assignment 4

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This assignment has 4 tasks.

**Exercise 1**. Consider the interpolation points

| $x$ | 0 | 2 | 3 |
|---|---|---|---|
| $y$ | 1 | 1 | $\frac{5}{2}$ |

**1.a)** Compute the interpolating polynomial using a linear combination of Lagrange polynomials. Check that this polynomial indeed interpolates the points.

**1.b)** Compute the value of $P(1)$ using the Neville algorithm. Compare with the value you get from the polynomial you previously computed.

**1.c)** Compute the interpolating polynomial using Newton's divided differences. Compare with the solution you already know.

**Exercise 2**. Given the interpolation points $x_0, \ldots, x_n$, one defines the corresponding Lagrange polynomials $\ell_0, \ldots, \ell_n$ as the unique polynomial such that

$$\ell_k(x_j) = \begin{cases} 1 & \text{if } k = j \\ 0 & \text{otherwise} \end{cases}$$

**2.a)** Recall the expression of the Lagrange polynomial $\ell_k$

**2.b)** Show (without calculation) that

$$\ell_0 + \ell_1 + \cdots + \ell_n = 1$$

**2.c)** Show (without calculation) that

$$x_0\ell_0(x) + x_1\ell_1(x) + \cdots + x_n\ell_n(x) = x$$

**Exercise 3**. You should follow this instructions on the Python installation page of the course. In order to plot a function in the interval $[0, 1]$ with 500 points, for example the function $x \mapsto x^2$, you may use

```
xs = linspace(0.,1.,500)
ys = xs**2
plot(xs,ys)
```

Try that first.

**3.a)** Plot between 0.995 and 1.005 the function $x \to (x-1)^6$ calculated by

```
ys = (xs - 1)**6
```

**3.b)** Do the same with the *same* function but expanded as $1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6$

What do you observe? How do you explain that?

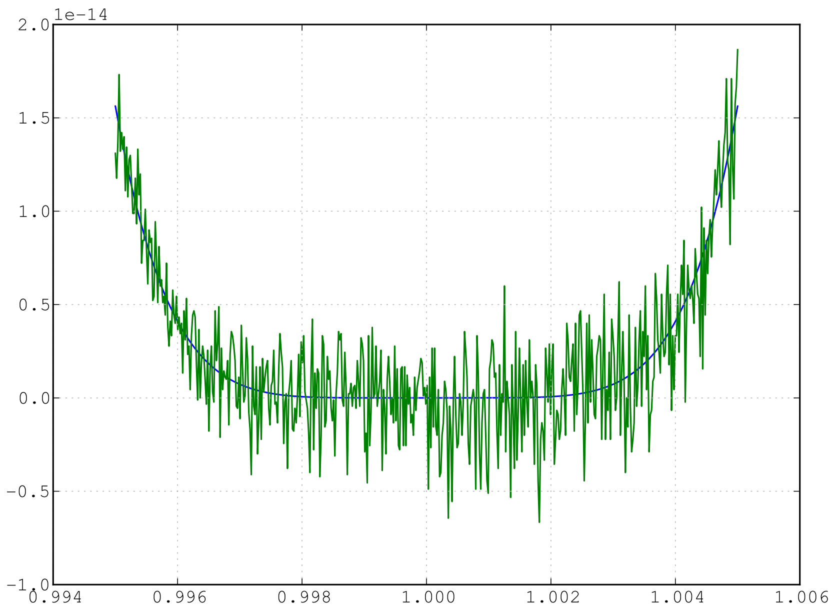
**Exercise 4**. Consider the functions $T_n(x)$, defined on $[-1, 1]$ by

$$T_n(x) = \cos(n \cdot \arccos(x)), \qquad \text{for } n = 0, 1, 2, \ldots.$$

Show that they satisfy the recursion formula

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_{n+1} = 2xT_n(x) - T_{n-1}(x).$$

Conclude that the function $T_n$ are in fact *polynomials*. (Hint: use the auxiliary variable $x = \cos(\theta)$)


1e-14
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0.994
0.996
0.998
1.000
1.002
1.004
1.006