



MA2501 Numeriske Metoder
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Training Assignment 5

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This assignment has 5 tasks.

Exercise 1. 1.a) Suppose that p interpolates $(x_0, y_0), \dots, (x_n, y_n)$ and q interpolates $(x_0, z_0), \dots, (x_n, z_n)$. Express the polynomial that interpolates $(x_0, 3y_0 - 2z_0), \dots, (x_n, 3y_n - 2z_n)$ using p and q .

1.b) Suppose that p interpolates

$$\begin{array}{c|cccc} x & 1 & 2 & 4 & 5 \\ \hline y & 3 & 4 & 2 & 8 \end{array}$$

and that q interpolates

$$\begin{array}{c|cccc} x & 1 & 2 & 3 & 5 \\ \hline y & 3 & 4 & 5 & 8 \end{array}$$

Express the polynomial that interpolates

$$\begin{array}{c|cccccc} x & 1 & 2 & 3 & 4 & 5 \\ \hline y & 3 & 4 & 5 & 2 & 8 \end{array}$$

in terms of p and q . Verify that the new polynomial indeed interpolates the new points.

Exercise 2. Suppose that two polynomials p_1 and p_2 of degree 3 both interpolate the same points $(x_0, y_0), (x_1, y_1), (x_2, y_2)$. Show that there exists a scalar λ such that

$$p_1 = p_2 + \lambda(x - x_0)(x - x_1)(x - x_2).$$

Exercise 3. Pick a polynomial P of degree k (considered as a function that we are going to interpolate), and pick n distinct points x_0, \dots, x_{n-1} .

3.a) Show that

$$P(x) = P[x_0] + (x-x_0)P[x_0, x_1] + \dots + (x-x_0) \cdots (x-x_{k-1})P[x_0, \dots, x_k]$$

3.b) Show that $P[x_0, \dots, x_m]$ is zero whenever $m > k$.

Exercise 4. Show that $f[x_0, \dots, x_n]$ does not depend on the order of the interpolation points. For instance, $f[x_0, x_1, x_2, x_3] = f[x_3, x_2, x_1, x_0]$. (Hint: use the definition of $f[x_0, \dots, x_{n-1}]$ as the highest order coefficient of the corresponding interpolating polynomial)

Exercise 5. This is a programming task. Note that you can obtain the length of a list using `len`, so `len(L)` is the length of the list L .

5.a) Write a function that takes a list as argument, and prints its elements one by one.

5.b) Write a function that takes a list as argument, and returns the sum of its values.

5.c) Now let us try to program Neville's algorithm. What arguments should the corresponding function need?

5.d) Try to express what you do manually for Neville's algorithm in a very systematic way, that is, column by column, line by line, how many calculations are there for each columns, etc.

5.e) Try to implement Neville algorithm, and test it!