Norwegian University of Science and Technology
Department of Mathematical Sciences

MA2501 Numeriske Metoder
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Training Assignment 6

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This assignment has 3 tasks.

Exercise 1. What is the order of the formula

$$\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{2h^2}$$

for approximating $f''(x_0)$?

Exercise 2. Consider again the formula

$$\varphi(h) := \frac{f(x_0+h)-f(x_0)}{h}.$$

2.a) Construct a new formula $\xi(h)$ by taking the value at zero of the following interpolation points: $(1/4, \varphi(h/4)), (1/2, \varphi(h/2)), (1, \varphi(h))$. You may use Neville's algorithm to achieve that.

2.b) Show, using a Taylor expansion of f at x_0, that $\xi(h)$ approximates $f'(x_0)$ at order three

2.c) Plot the error $|\xi(h) - f'(x_0)|$ versus $\ln(h)$ for a function f and a point x_0 of your choice. Does that confirm that the order is three?

Exercise 3. Consider the equation system

$$\begin{aligned} x_1 + x_2 &= 2 \\ \alpha x_1 + x_2 &= 2 + \alpha. \end{aligned}$$

For which values of α will naive Gauss elimination (that is, without any row permutation) will give a wrong answer? Try to explain what will happen in the computer.