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MA2501 Numeriske Metoder
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Training Assignment 8

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This assignment has 4 tasks.

Exercise 1. Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & -1 & 3 \\ 1 & -2 & 2 & 4 \\ -2 & 4 & -2 & 2 \\ -1 & 6 & -9 & 5 \end{bmatrix}.$$

1.a) Change the order of the rows to obtain a new matrix A' such that

$$A' = LU$$

with L lower triangular with ones on the diagonal, U upper triangular, and the coefficients in the lower triangular matrix L are lower than or equal to one.

1.b) Find a permutation matrix P such that

$$A = PA',$$

so that we obtain the final decomposition $A = PLU$.

1.c) Use the `lu` function in Python or Matlab on the matrix A. Do you obtain the same result?

1.d) Use your decomposition PLU to solve the problem

$$Ax = b$$

where

$$b = (1, -1, 6, -1)$$

Exercise 2. Consider the problem

$$Ax = b$$

where

$$A = \begin{bmatrix} 6 & -1 & -1 \\ -1 & 9 & -2 \\ -2 & -1 & 8 \end{bmatrix} \qquad b = \begin{pmatrix} 4 & 6 & 5 \end{pmatrix}.$$

Compute the first two steps of the Jacobi and Gauss-Seidel methods for that problem (either by hand or with a computer), with the initial value $x_0 = (0, 0, 0)$.

Exercise 3. Suppose that for a matrix T we have

$$\max_{\|z\|=1} \|Tz\| < 1.$$

Show that the fixed point iteration method $x_{n+1} = Tx_n + c$ is convergent (for any fixed vector c and any initial value x_0).

Exercise 4. We want to solve numerically the problem

$$\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} + u = 1$$

on the interval $[0, 1]$. After discretization with N points in the interval $[a, b]$, the problem is reduced to the linear problem

$$A_N x = b_N.$$

The matrix A_N has size $N \times N$ and has $2N^2 + 1$ on the diagonal, and $-N^2$ on the upper and lower diagonal:

$$A_N = \begin{bmatrix} 2N^2+1 & -N^2 & & \\ -N^2 & \ddots & \ddots & \\ & \ddots & \ddots & -N^2 \\ & & -N^2 & 2N^2+1 \end{bmatrix}$$

4.a) Write down one step of the Jacobi and Gauss-Seidel methods.

4.b) Write down the relation between the error at step $n+1$ as a function of the error at step n.

4.c) Show directly that the Jacobi method converge. What happens when N becomes very large?