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MA2501 Numeriske Metoder
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Training Assignment 10

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This assignment has 4 tasks.

Exercise 1. Suppose that we construct a quadrature formula (with nodes c_i and weights w_i) The corresponding integration formula is thus

$$I_h(f) = \sum_{k=0}^{N-1} h \sum_{i=1}^{s} w_i f(a_k + c_i h),$$

where $a_0 = a$, $a_N = b$, and $h = (b-a)/N$. Suppose that that the quadrature formula does not integrate constants exactly, i.e.,

$$\sum_{i=1}^{s} w_i \neq 1.$$

Show that the integration formula does not converge to the integral of f, i.e., in general

$$\lim_{h\to 0} I_h(f) \neq \int_a^b f(x)\,\mathrm{d}x.$$

Is that in agreement with the order formula derived in the lecture?

Exercise 2. **2.a)** Recall what the Vandermonde matrix is and what it was used for

2.b) Choose quadrature nodes $c_1, \dots, c_s$ in the interval $[0, 1]$. The corresponding weights are chosen such that the quadrature is exact for polynomials of degree $s - 1$. Show that the vector w containing the corresponding weights, i.e., $w = (w_1, \dots, w_s)$, is the solution of the linear system

$$V^{\mathsf{T}} w = b,$$

where the vector b is

$$b = (1, 1/2, \dots, 1/s)$$

and V is the Vandermonde matrix for the points $c_1, \dots, c_s$.

Exercise 3. **3.a)** Compute the first three Legendre polynomials p_0, p_1 and p_2, by orthogonalising the polynomials 1, x, x^2 with respect to the scalar product

$$\langle p, q \rangle = \int_{-1}^{1} p(x) q(x) \, \mathrm{d}x$$

3.b) Compute the roots of the polynomial p_2. Compute the weights of the corresponding formula (hint: apply the formula to the polynomial 1 and x that are integrated exactly)

3.c) Check that this integration formula integrates exactly polynomials of degree lower than or equal to 3, but not 4.

3.d) What is the order of the corresponding integration formula?

Exercise 4. There is a recursion relation between the Legendre polynomials, the goal is to find it out.

4.a) Show that the polynomial xp_k is orthogonal to all the polynomials of degree less than or equal to $k - 2$.

4.b) Expand xp_k in the basis $p_0, \dots, p_{k+1}$ to find the recurrence relation