Norwegian University of Science and Technology
Department of Mathematical Sciences

MA2501 Numeriske Metoder
Olivier Verdier

Training Assignment 12

2012-04-12

This assignment has 4 tasks.

Exercise 1. Consider the following differential equation:

$$u'(t) = -u(t)\cos(t)$$

1.a) Try to understand what is the unknown in that equation. Write one possible initial condition.

1.b) Write down one step of the Explicit and Implicit Euler methods for that differential equation

Exercise 2. Consider Newton's Equation modelling an oscillator without friction.

$$u''(t) = -u(t)$$

2.a) Write down one step of the Explicit Euler method to solve that equation numerically.

2.b) Do the same with the Runge Kutta 4 method (p. 443 in C&K).

Exercise 3. Given a numerical method, for instance explicit Euler, one may define the corresponding "flow" as a mapping:

$$\Phi_h : \ u_0 \longmapsto u_1$$

For instance, in the explicit Euler case, this mapping is given by

$$\Phi_h(u_0) = u_0 + hf(u_0)$$

The *adjoint method* corresponding to a given flow is given by the flow

$$\Psi_h := (\Phi_{-h})^{-1}$$

3.a) What is the adjoint method corresponding to explicit Euler?

3.b) What is the adjoint method of the Trapezoidal rule, given by:

$$\Phi_h(u_0) = u_1 \quad \text{such that} \quad u_1 - u_0 = h(f(u_0) + f(u_1))/2$$

Exercise 4. In this exercise, we compute the stability region of the trapezoidal rule.

4.a) Apply the trapezoidal rule to the differential equation $u' = \lambda u$, and write the result as

$$u_1 = A(\lambda h)u_0$$

where A is an expression that you will compute.

4.b) What is the region of the complex plane corresponding to the equation

$$|A(z)| \leq 1$$

What restriction is there to the step size h if $\Re(\lambda) < 0$ (stable system)?