**Norwegian University of Science and Technology**
**Department of Mathematical Sciences**



**MA2501** Numeriske Metoder
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# Project

2011-02-16

- This project is mandatory, and counts for the final grade.
- You may work in groups of at most two people
- You have to produce a short report with your solutions (preferably using LaTeX).
- The report, and the code produced, should be sent electronically to olivier.verdier@math.ntnu.no
  at the latest on **Thursday 8 March at 17:00**.

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**Problem 1**. Pick a polynomial $P$ of degree 3. Do *not* choose a polynomial you find on the internet, just find a new polynomial of your own. Choose an interval $[a, b]$ of your choice. You should pick an interval of you own, like $[2, 3.5]$, or something of that kind, *not* $[0, 1]$ or $[-1, 1]$. Choose distinct interpolations points $x_0, \dots, x_k$ with $k = 3$, such that two of the interpolation points are at the boundary of the interval.

**1.a)** Pick one of the interpolation points, and write the corresponding Lagrange polynomial.

**1.b)** Pick a point which is not an interpolation point, and compute the value of the interpolation polynomial at that point using the Neville algorithm.

**1.c)** Compute the Newton polynomials which interpolate $P$ at the points

- $x_2$
- $x_2, x_3$
- $x_2, x_3, x_1$
- $x_2, x_3, x_1, x_0$

Try to minimize the number of calculations!

**1.d)** Write a function that takes a function $f$ and a list of points $x_0, \ldots, x_k$ as arguments, and returns a list of the function's value at those points $f(x_0), \ldots, f(x_k)$.

**1.e)** Look at the documentation of `polyfit` and `polyval` to see how to compute the interpolation polynomial. Use those functions to plot the interpolation polynomials of **1.c**. Plot also the original polynomial $P$.

**1.f)** Now consider Chebyshev interpolation points for the interval $[a, b]$ that you have chosen. Program a function that computes and stores $k$ Chebyshev points in an interval $[a, b]$ (for any value of $k$), and returns them in a list.

**1.g)** Repeat **1.e** but now with the Chebyshev points replacing the interpolation points $x_0, \ldots, x_3$.

**1.h)** Use the interpolation error formula to estimate the error when you interpolate $P$ with three interpolation points (of your choice). (You may plot the appropriate functions to estimate their maximum value)

Now estimate the *actual* error between the interpolation polynomial and the function $P$. How does it compare to the previous error estimation?