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MA2501 Numeriske Metoder
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Training Assignment 3

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This assignment has 6 tasks.

Exercise 1. 1.a) Find the Lagrange form of the interpolation polynomial of the lowest possible degree that interpolates the table

x	0	2	3	4
y	7	11	28	63

1.b) Verify that the polynomials

$$p(x) = 5x^3 - 27x^2 + 45x - 21$$
$$q(x) = x^4 - 5x^3 + 8x^2 - 5x + 3$$

both interpolate the points in the table

x	1	2	3	4
y	2	1	6	47

Why doesn't this contradict the uniqueness theorem?

Exercise 2. Use the "clever formula"

$$\frac{x - x_n}{x_0 - x_n} P(x) + \frac{x - x_0}{x_n - x_0} Q(x)$$

to construct the interpolating polynomial for the following data recursively:

That is, first interpolate constants on each points, then lines on pairs of points (two lines), then one quadratic polynomial.

x	0	2	3
y	7	11	28

Constants are $7, 11, 28, 63$. Lines are $\frac{x-2}{-2}7+\frac{x}{2}11$ and $\frac{x-3}{-1}11+\frac{x-2}{1}28$. Quadratic polynomial is then

$$\frac{x-3}{-3}\left(\frac{x-2}{-2}7+\frac{x}{2}11\right)+\frac{x}{3}\left(\frac{x-3}{-1}11+\frac{x-2}{1}28\right)$$

Exercise 3. Predict the result of the following code:

```
x = (1e20 + 1) - 1e20
print x
```

Exercise 4. Program Newton's method with trust region in one dimension (you may use the function `min`, which returns the minimum of two numbers, and the function `abs`, which returns the absolute value of a number).

```
for i in range(100):
  dx = x - (x**2 - 1)/(2*x)
  x = x + min(T/abs(dx), 1)*dx
```

Exercise 5. Consider Newton's method in two dimensions, i.e.,

$$F'(X_n)\cdot(X_{n+1}-X_n)=-F(X_n).$$

Put it in a fixed point iteration form, i.e., in the form

$$X_{n+1}=G(X_n)$$

and compute the Jacobian $G'(\bar{x})$ at a root of F, i.e., at a point $\bar{x}$ such that $F(\bar{x})=0$.

$$G(X) := X-(F'(X))^{-1}F(X)$$
$$A(X) := (F'(X))^{-1}F(X)$$

we have

$$A'(\bar{X})=\mathbb{I}$$

So

$$G'(\bar{X})=0$$

Exercise 6. What is the condition on the function $G\colon \mathbf{R}^2 \to \mathbf{R}^2$ for the fixed point iteration

$$X_{n+1} = G(X_n)$$

to converge (provided that the initial value X_0 is close enough to a fixed point $\bar{X}$)?