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MA2501 Numeriske Metoder
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Training Assignment 5

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This assignment has 5 tasks.

Exercise 1. 1.a) Suppose that p interpolates $(x_0, y_0), \dots, (x_n, y_n)$ and q interpolates $(x_0, z_0) \dots (x_n, z_n)$. Express the polynomial that interpolates $(x_0, 3y_0 - 2z_0), \dots, (x_n, 3y_n - 2z_n)$ using p and q.

Clearly, $3p - 2q$ interpolates the new set of points.

1.b) Suppose that p interpolates

x	1	2	4	5
y	3	4	2	8

and that q interpolates

x	1	2	3	5
y	3	4	5	8

Express the polynomial that interpolates

x	1	2	3	4	5
y	3	4	5	2	8

in terms of p and q. Verify that the new polynomial indeed interpolates the new points.

Using Neville's formula, we obtain that

$$(x - 3)p - (x - 4)p$$

interpolates the new set of points.

Exercise 2. Suppose that two polynomials p_1 and p_2 of degree 3 both interpolate the same points $(x_0, y_0), (x_1, y_1), (x_2, y_2)$. Show that there exists a scalar λ such that

$$p_1 = p_2 + \lambda(x - x_0)(x - x_1)(x - x_2).$$

The polynomial $p_1 - p_2$ has a coefficient λ in front of x^3. As a result, $p_1 - p_2 - \lambda(x - x_0)(x - x_1)(x - x_2)$ has degree 2, and is zero on the three distinct points x_0, x_1, x_2, so it is zero.

Exercise 3. Pick a polynomial P of degree k (considered as a function that we are going to interpolate), and pick n distinct points $x_0, \ldots, x_{n-1}$.

3.a) Show that

$$P(x) = P[x_0] + (x - x_0)P[x_0, x_1] + \cdots + (x - x_0)\cdots(x - x_{k-1})P[x_0, \ldots, x_k]$$

Uniqueness of the interpolating polynomial.

3.b) Show that $P[x_0, \ldots, x_m]$ is zero whenever $m > k$.

Clear from expressing that $P - Q = 0$ where Q is the Newton polynomial interpolating P at the points $x_0, \ldots, x_{n-1}$.

Exercise 4. Show that $f[x_0, \ldots, x_n]$ does not depend on the order of the interpolation points. For instance, $f[x_0, x_1, x_2, x_3] = f[x_3, x_2, x_1, x_0]$. (Hint: use the definition of $f[x_0, \ldots, x_{n-1}]$ as the highest order coefficient of the corresponding interpolating polynomial)

Exercise 5. This is a programming task. Note that you can obtain the length of a list using `len`, so `len(L)` is the length of the list L.

5.a) Write a function that takes a list as argument, and prints its elements one by one.

```
def f(x):
  for i in range(len(x)):
    print x[i]
```

5.b) Write a function that takes a list as argument, and returns the sum of its values.

```
def f(x):
  s = 0
  for i in range(len(x)):
    s = s + x[i]
  return s
```

5.c) Now let us try to program Neville's algorithm. What arguments should the corresponding function need?

> We need two lists, one for the x values, one for the y values, and a third argument corresponding to the point x_0 for which we want $P(x_0)$.

5.d) Try to express what you do manually for Neville's algorithm in a very systematic way, that is, column by column, line by line, how many caluculations are there for each columns, etc.

5.e) Try to implement Neville algorithm, and test it!