**Norwegian University of Science and Technology**
**Department of Mathematical Sciences**



**MA2501** Numeriske Metoder
Olivier Verdier

# Training Assignment 6

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This assignment has 3 tasks.

**Exercise 1**. What is the order of the formula

$$\frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{2h^2}$$

for approximating $f''(x_0)$?

**Exercise 2**. Consider again the formula

$$\varphi(h) := \frac{f(x_0+h) - f(x_0)}{h}.$$

**2.a)** Construct a new formula $\xi(h)$ by taking the value at zero of the following interpolation points: $(1/4, \varphi(h/4)), (1/2, \varphi(h/2)), (1, \varphi(h))$. You may use Neville's algorithm to achieve that.

The expression from Neville's algorithm is

$$\xi(h) = \frac{4}{3}(2\varphi(\frac{h}{4}) - \varphi(\frac{h}{2})) - \frac{1}{3}(2\varphi(\frac{h}{2}) - \varphi(h)).$$

This leads to

$$\xi(h) = \frac{8\varphi(\frac{h}{4}) - 6\varphi(\frac{h}{2}) + \varphi(h)}{3},$$

which we may rewrite as

$$\xi(h) = \frac{8f(x_0+\frac{h}{4}) - 6f(x_0+\frac{h}{2}) + f(x_0+h) - 3f(x_0)}{3h}.$$

**2.b)** Show, using a Taylor expansion of $f$ at $x_0$, that $\xi(h)$ approximates $f'(x_0)$ at order three

**2.c)** Plot the error $|\xi(h) - f'(x_0)|$ versus $\ln(h)$ for a function $f$ and a point $x_0$ of your choice. Does that confirm that the order is three?

**Exercise 3**. Consider the equation system

$$\begin{aligned} x_1 + x_2 &= 2 \\ \alpha x_1 + x_2 &= 2 + \alpha. \end{aligned}$$

For which values of $\alpha$ will naive Gauss elimination (that is, without any row permutation) will give a wrong answer? Try to explain what will happen in the computer.