Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4310 Optimal control of PDEs Spring 2015

Exercise set 2

Last week's exercises demonstrate at least two things: (1) optimal controls (or at least minimizing sequences of controls) may be rather irregular (e.g., discontinuous or worse), which in turn leads to (2) the state (differential) equations may not be understood in the classical sense, as the control enters them.

We will look at a mathematical setup which can accommodate this situation. We begin with a review of the relevant function spaces.

Reading:

Review normed/L^p spaces in [Tröltzsch] (Section 2.1-beginning of Section 2.2) and linear mappings/functionals (Section 2.4.1). Begin reading about Sobolev spaces (Sections 5.1-5.2 in [Evans]).

Note: contact me if you need copies of Chapter 5 from [Evans].

Exercises:

- [Tröltzsch], 2.2, 2.3, 2.5, 2.6.
- [Evans], Chapter 5: 7, 1.
- Arzela–Ascolli theorem states that a set $S \subset C[a,b]$ is *relatively compact* (i.e., its closure is compact) if and only if it is bounded and and equicontinuous. Show that for every $\gamma > 0$, every bounded set S in $C^{0,\gamma}[a,b]$ is relatively compact set in $C[a,b]$ (one says that $C^{0,\gamma}[a,b]$ is compactly embedded into $C[a,b]$).