Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4310 Optimal control of PDEs Spring 2015

Exercise set 7

We finally get to the systematic derivation of first order optimality conditions and some numerical methods.

Reading:

Sections 2.10, 2.12-2.12.2, 2.12.4 in [Tr]. Skim-read the rest of Section 2.12.

Recommended exercises:

1. Exercise 2.19
2. Implement the FVM discretization (see below) of conditioned gradient and projected gradient methods for the optimal heat source problem. Verify the correctness of your implementation on the test example constructed in Section 2.9.1. Compare the performance of both methods.

Cell-based FVM on orthogonal polyhedral meshes

The discretization scheme is exceedingly simple: the domain is subdivided into disjoint polyhedral cells. Within each cell, all functions are approximated with a single constant (say, an average value of a function within this cell, or its value within the center of the cell). As a result, volumetric integrals are approximated as follows:

$$\int_{\Omega} f(x)\,\mathrm{d}x \approx \sum_{c} f_c v_c,$$

where the summation is over all cells in the discretization, f_c is the constant approximation of f in the cell c, and v_c is the volume (area in 2D) of the cell c. We omit the details about how the governing PDE is discretized; the commented code implementing the discretization of the Laplace problem can be downloaded from the wiki page.