Norwegian University of Science and Technology
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TMA4310 Optimal control of PDEs
Spring 2015

Exercise set 8

We briefly touch upon the question of higher regularity of optimal solutions/control to linear-quadratic elliptic control problems. From next time on we will look into control of non-linear systems.

Reading:

Sections 2.14–2.15 in [Tr]; also start with the non-linear elliptic control by reading Section 4.1.

Recommended exercises:

1. In the proof or Theorem 2.37 (and later also in Theorem 2.38) it is claimed that the fact that $\mathbb{P}_{[u_a,u_b]}$ maps $H^1(\Omega)$ continuously into itself follows from the fact that $|\cdot|$ maps $H^1(\Omega)$ continuously into itself. Assuming the latter, provide the details of this assertion.

2. Exercise 5.17 (i) in [Evans]: let Ω be a bounded domain, $1 \le p < +\infty$. Show that if $u \in W^{1,p}(\Omega)$ then also $|u| \in W^{1,p}(\Omega)$. Hint: one could for example utilize that $|u| = \lim_{\varepsilon\to 0}[\varepsilon^2 + u^2]$ and prove the assertion for $C^\infty(\Omega)$ functions, which are dense in $W^{1,p}(\Omega)$.

3. In addition to the assumptions of Theorem 2.38 assume that Ω is a bounded domain of the class $C^{1,1}$. Utilise the appropriate results quoted in Section 2.14 to assert that $y, p \in H^2(\Omega)$.

4. In the assumptions of the previous exercise, assume further that $\beta \in C^\infty(\bar{\Omega})$. Can we conclude that the optimal control $u \in H^2(\Omega)$? Hint: try to construct a simple counterexample to the assertion that $|\cdot|$ maps $H^2(\Omega)$ into $H^2(\Omega)$.