Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4310 Optimal control of PDEs Spring 2015

Exercise set 9

The goal now is to repeat the same program (existence of states/optimal controls/optimality conditions/numerical methods) for the control of semi-linear elliptic PDEs. We begin with the existence of solutions.

Reading:

Section 4.2 in [Tr]. Some of the results are proved in Section 7.2 (or in external references cited in [Tr]), which you are encouraged to consult if you are curious about all the details.

Recommended exercises:

1. Exercises 4.1, 4.2, 4.6 in [Tr].
2. Note that monotonic growth of the non-linear part in the governing PDE is essential (see Assumption 4.2 (iii)). In view of examples (4.13) and (4.14), consider a one-dimensional Dirichlet problem on the interval $0 < x < 1$

$$-y''(x) + d(x, y(x)) = f(x),$$

with the homogeneous boundary conditions $y(0) = y(1) = 0$. Show that it admits infinitely many solutions with $f(x) \equiv 0$ and $d(x, y) = -\pi^2 y$. Show that the same conclusion remains valid if the homogeneous Dirichlet boundary conditions are replaced with homogeneous Neumann boundary conditions.