CURRICULUM FOR MA3201 FALL 2008

Chapters

9, 10, 14: 1-5, 19: 1-3, 20, 21.

Comments

Chapter 9.

(1) Any ring R by definition has identity element 1, and any subring S of R has by definition the same identity element as R.

(2) Additional examples of algebras: **Path algebras of quivers over a field**.

Let F be a field and Q a finite quiver, that is, a finite set of vertices and a finite set of arrows between vertices.

Example.

$$Q : 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3$$

The path algebra FQ has as vector space over F the paths in Q, including the "trivial" paths, as basis. In the example we then have as basis the arrows α, β, the composition $\beta\alpha$ and the 3 trivial paths e_1, e_2 and e_3 associated with the 3 vertices. Multiplication of two basis elements is given by composition of paths if possible, and is defined to be 0 otherwise. To illustrate, in the above example we have $\beta \cdot \alpha = \beta\alpha$, $\alpha \cdot \beta = 0$, $\alpha \cdot e_1 = \alpha$ and $e_1 \cdot \alpha = 0$.

The elements in FQ are then linear combinations of the paths, with coefficients in F. So if $p_1, \dots, p_n$ are paths in Q, then the elements of FQ are of the form $\sum_{i=1}^n a_i p_i$, where $a_i \in F$ for $i = 1, \dots, n$. Then $(\sum_{i=1}^n a_i p_i)(\sum_{i=1}^n a'_i p_i) = \sum_{1 \leq i,j \leq n} a_i a'_j p_i p_j$. One can then show that FQ is finite dimensional over F if and only if the quiver Q has no oriented cycles.

If $1, \dots, n$ are the vertices in Q, then the elements $e_1, \dots, e_n$ are idempotent elements and $1 = e_1 + \dots + e_n$ is the identity element. For $R = FQ$ we have $R = Re_1 \oplus \cdots \oplus Re_n$, where Re_i is a subspace of R with the paths starting at i as basis.

In the example, the subspace spanned by α, β and $\beta\alpha$ is an ideal I which is nilpotent, and we have $R/I \simeq F \times \cdots \times F$ (n copies).

p.163: An integral domain is by definition commutative.

Chapter 14.

P. 251: Let M, N be modules over a ring R. We have *external direct sum* $M \oplus N$ of M and N, where the elements are pairs (m, n) for $m \in M$, $n \in N$, $(m, n) + (m', n') = (m + m', n'n')$ and for $r \in R$, then $r(m, n) = (rm, rn)$.

P. 260: We use *semisimple module* instead of *completely reducible module*. Add an extra statement to Theorem 3.6: R is a semisimple R-module.

Chapter 19.

P. 368: We do not deal with finitely cogenerated modules.

P. 370: Usually a ring R is said to be noetherian if it is both left noetherian and right noetherian.

Chapter 20.

(1) Consult chapter 11.1 for properties of UFDs needed in the text.
(2) Note a typo on page 394. The elements in a row or column can be multiplied with an *invertible* element.

Chapter 21.

(1) Let V be an n-dimensional vector space over F and $T : V \to V$ a linear transformation. Then the minimal polynomial of T is a nonzero monic polynomial $p(x)$ of minimal degree such that $p(T) = 0$. Then $p(x)|c(x)$, where $c(x)$ is the characteristic polynomial of T, and we have $c(T) = 0$ (Cayley-Hamilton). Note that when $f_1(x)|\cdots|f_r(x)$ are the monic invariant factors of T, then $p(x) = f_r(x)$ and $f_1(x)\cdots f_r(x) = (-1)^n c(x)$.
(2) In section 5 we only discussed the case $F = \mathbb{C}$.