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Contact during midterm exam:
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MIDTERM EXAM IN MA3201 RINGS AND MODULES

Friday, October 5, 2007
Time: 12:15 – 14:00, F3
Permitted aids: None.
English

You must give arguments for all your answers.

Problem 1

Let S be the continuous functions from $[0, 1]$ to $\mathbf{R}$ (real numbers), where $(f+g)(x) = f(x)+g(x)$ and $(fg)(x) = f(x)g(x)$, for f, g i S and $x \in [0, 1]$. Then S is a commutative ring. (Should not be shown.)

a) Show that $I = \{f \in S; f(1/2) = 0\}$ is an ideal in S

b) Find some $f \neq 0$ in S which is a zero divisor.

c) Show that the factor ring S/I is isomorphic to the ring $\mathbf{R}$.

Problem 2

Let F be a field and R the ring $\left(\begin{smallmatrix} F & F \\ F & F \end{smallmatrix}\right)$.

a) Show that $I = \left(\begin{smallmatrix} F & 0 \\ F & 0 \end{smallmatrix}\right)$ is a minimal left ideal in $\mathbf{R}$.

b) Show that the sum $R = \left(\begin{smallmatrix} F & 0 \\ F & 0 \end{smallmatrix}\right) + \left(\begin{smallmatrix} 0 & F \\ 0 & F \end{smallmatrix}\right)$ of left ideals is a direct sum.

c) Is R a semisimple R-module? (semisimple = completely reducible).

Problem 3

Let $\mathbf{Z}$ be the ring of integers.

a) Show that $\mathbf{Z}$ has no simple submodules.

b) Is $\mathbf{Z}$ a semisimple $\mathbf{Z}$-module?

c) Is $\mathbf{Z}$ a free $\mathbf{Z}$-module?

Problem 4

Find all left ideals in the ring $R = \left(\begin{smallmatrix} \mathbf{Z}_2 & 0 \\ \mathbf{Z}_2 & \mathbf{Z}_2 \end{smallmatrix}\right)$