MA3201 - Problem sheet 4
fall 2008

The problem session will be on Thursday October 23rd in F3.

Problems from the book:

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381	6
388	1*

Also: All problems from the midterm from last year.

Problem 1. Let R be a ring and $\phi : \mathrm{Hom}_R(R, R)^{op} \to R$ given by $\phi(g) = g(1)$. Show that ϕ is a ring isomorphism.

Problem 2. Let R be a ring, $I \subseteq R$ an ideal, M an R/I-module, and hence an R-module by $rm = (r + I)m$ for $r \in R$, $m \in M$. Let $N \subseteq M$ be a subgroup. Show that N is an R-submodule if and only if N is an R/I-submodule.

Problem 3*. Let $R = \mathbb{Z}_2 G$, where G is a group of order 2. Show that R is not a semisimple ring.

Problem 4. Let Q be the quiver

$$1 \to 2 \to 3$$

and let F be a field. Find a nilpotent ideal I in the path algebra FQ such that FQ/I is a semisimple ring.

Problem 5*. Let F be a field, and let

$$R = \left\{ \begin{pmatrix} a & 0 & 0 \\ b & \alpha & 0 \\ c & 0 & \alpha \end{pmatrix} \mid a, b, c, \alpha \in F \right\}.$$

(a) Show that R is a ring with 1. Is R commutative?
(b) Let $J = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ b & 0 & 0 \\ c & 0 & 0 \end{pmatrix} \mid b, c \in F \right\}$. Show that J is a two-sided ideal in R, and that $R/J \simeq F \times F$.
(c) Let $e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Show that Re is a maximal left ideal in R, og find all left ideals in R which are contained in Re.

Problems marked with * can be handed in at the lecture Monday October 20th.