MA3201 Rings and Modules, 2014

Problem Sheet 1

To be discussed on Friday 5 September and Friday 12 September.

Problems from: Bhattacharya, P. B.; Jain, S. K.; Nagpaul, S. R. Basic abstract algebra. Second edition. Cambridge University Press, Cambridge, 1994.

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Problem 1. Let R be a ring. Find the centre of the ring $M_2(R)$.

Problem 2. Let $\mathbb{F}$ be a field. Let I be the subset:

$$I = \left\{ \begin{pmatrix} x & x \\ y & y \end{pmatrix} : x, y \in \mathbb{F} \right\}$$

of $M_2(\mathbb{F})$. Show that I is a left ideal of $M_2(\mathbb{F})$. Is I a subring of $M_2(\mathbb{F})$?

Problem 3. Let $\mathbb{F}$ be a field. Let Q be the quiver:

$$1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3$$

and $R = \mathbb{F}Q$ the corresponding path algebra. Show that the subspace I of R spanned by $\alpha, \beta, \beta\alpha$ is an ideal of R.

Problem 4. Let R and S be rings. Show that the left ideals of the direct product $R \times S$ are all of the form

$$I \times J = \{(x, y) : x \in I, y \in J\},$$

where I is a left ideal of R and J is a left ideal of S.

R. J. Marsh, 20/8/14.