MA3201 Rings and Modules, 2014

Problem Sheet 2

To be discussed on Friday 19 September and Friday 26 September.

Problems from: Bhattacharya, P. B.; Jain, S. K.; Nagpaul, S. R. Basic abstract algebra. Second edition. Cambridge University Press, Cambridge, 1994.

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Problem 1. Let R be a commutative ring and P a prime ideal of R. Show that R/P is an integral domain.

Problem 2. Let $\mathbb{F}$ be a field and let

$$R = U_3(\mathbb{F}) = \begin{pmatrix} \mathbb{F} & \mathbb{F} & \mathbb{F} \\ 0 & \mathbb{F} & \mathbb{F} \\ 0 & 0 & \mathbb{F} \end{pmatrix}.$$

Show that

$$I = \begin{pmatrix} 0 & \mathbb{F} & \mathbb{F} \\ 0 & 0 & \mathbb{F} \\ 0 & 0 & 0 \end{pmatrix}$$

is an ideal of R and find the ideals of R containing I. Which of these are maximal ideals of R?

Problem 3.

Let n be an integer, $n \geq 2$. Show that the ring $\mathbb{Z}$ cannot be written as a direct sum of n non-zero ideals.

R. J. Marsh, 09/09/14.