MA3201 Rings and Modules, 2014

Problem Sheet 6

To be discussed on Friday 14 November and Friday 21 November.

Problems from: Bhattacharya, P. B.; Jain, S. K.; Nagpaul, S. R. Basic abstract algebra. Second edition. Cambridge University Press, Cambridge, 1994.

Page	Problem numbers
401	1, 3(b)
409	(c)

Problems from old exams:

Exam	Problem number	Note
2013	1	Part (c) is a challenge question.
2011	1	
2009	2	

Problem 1. For the matrix A in part (a) of question 1 of page 401 of the book, find square matrices P and Q such that PAQ is in Smith normal form.

Problem 2. A ring R is said to be a *Euclidean domain* if there is a function $\varphi : R \to \mathbb{Z}$ satisfying:

(a) For all nonzero elements $a, b \in R$ satisfying $a|b$, we have $\varphi(a) \leq \varphi(b)$.

(b) For all $a, b \in R$ with b nonzero, there are elements q, r in R such that $a = qb + r$ and $\varphi(r) < \varphi(b)$.

Answer Problem 1 on page 219. Note that, in the book, $a|b$ can only hold if a, b are both nonzero. But the question is still correct with our definition, i.e. that, for $a, b \in R$, we say $a|b$ if there is $c \in R$ such that $b = ca$.

Problem 3. It is known that every Euclidean domain is a principal ideal domain (see Theorem 3.2 on page 218 of the book). Give a technique for reducing a matrix over a Euclidean domain to its Smith normal form using only elementary row and column operations (Challenge question).

R. J. Marsh, 3/11/14.