MA8105

Nonlinear PDEs and Sobolev spaces
Spring 2019

Exercise set 1

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$\boxed{1}$ Let $a, b \geq 0$, $\frac{1}{p} + \frac{1}{q} = 1$. Prove Young's inequality:

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}.$$

Hint: $ab = e^{\frac{1}{p}\ln a^p + \frac{1}{q}\ln b^q}$ + convexity of exponential.

$\boxed{2}$ Let $\varepsilon > 0$. Prove $ab \leq \varepsilon \frac{a^p}{p} + \frac{b^q}{q\varepsilon^{q-1}}$

$\boxed{3}$ Prove Hölder in ℓ^p: $|\sum_n x_n y_n| \leq \|x\|_p \|y\|_q$.

Hint: Use Young's inequality and/or Google.

$\boxed{4}$ Prove $\|x\|_\infty \leq \|x\|_p \leq \|x\|_q \leq \|x\|_1$ for $1 < p < q < \infty$.

$\boxed{5}$ Holden Ex 4 p. 34

Prove that ℓ^p and ℓ^q are dual.

Hint: See appendix in Holden for hints.