Norwegian University of Science and Technology
Department of Mathematical Sciences

MA8105
Nonlinear PDEs and Sobolev spaces
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Exercise set 13

$\boxed{1}$ Let $f \in L^2(\mathbb{R}^d)$ and let $u \in W^{1,2}(\mathbb{R}^d)$ be the weak solution of

$$u - \Delta u = f \quad \text{in} \quad \mathbb{R}^d.$$

Use finite differences to show that $u \in W^{2,2}(\mathbb{R}^d)$.

Hint: Follow the steps layed out in Example 49 in my lecture notes (see leftmost collumn). Here you also find the definition of a weak solution of this equation. OBS: A key step is to use Riesz representation theorem to prove that

$$\|u\|^2_{W^{1,2}} = |\int fu| \leq \|u\|_{W^{1,2}} \sup_{0 \neq \phi \in W^{2,2}} \frac{|\int f\phi|}{\|\phi\|_{W^{2,2}}} := \|u\|_{W^{1,2}} \|f\|_{(W^{1,2})'}.$$

Note that f defines an element in $(W^{1,2})'$ through $F(\phi) = \int f\phi$ - i.e. a regular distribution.