Norwegian University of Science and Technology
Department of Mathematical Sciences

MA8105

Nonlinear PDEs and Sobolev spaces
Spring 2019

Exercise set 2

$\boxed{1}$ (Holden Ex 4 p 34) Show that $(c_0)' = \ell^1$.

$\boxed{2}$ Show ℓ^∞ is not separable.

Hint: For any countable $\{x_i\} \subset \ell^\infty$, find $y \in \ell^\infty$ s.t. $\|x_i - y\|_\infty \geq 1$ for all i.

$\boxed{3}$ Show that D is dense in ℓ^p for $p \in [1, \infty)$ when

$$D = \Big\{ x = \{x_k\}_k \ : \ x_k \in \mathbb{Q}, \text{ finite number of } x_k \neq 0 \Big\}$$

Hint: The set can be seen as a countable union of countable sets (why?) and is hence countable (you do not need to prove this).

$\boxed{4}$ Let $x_n = \{x_{n,k}\}_n \in \ell^1$ be defined by $x_{n,k} = 1$ for $n = k$ and 0 otherwise.

a) Show that x_n does not converge weakly in ℓ^1.

b) Show that x_n converge weakly * in ℓ^∞. Explain what this convergence is and why it does not imply weak convergence in this case.

$\boxed{5}$ (Ex 2 in Holden p 34) Prove that the weak limit is unique.

$\boxed{6}$ Ex 5 in Holden p 34.