Norwegian University of Science and Technology
Department of Mathematical Sciences

MA8105
Nonlinear PDEs and Sobolev spaces
Spring 2019

Exercise set 4

1 Holden Ex 1 p 52.

2 Prove that $T \in D'$ continuous iff $T \in D'$ continuous at 0.

3 Prove that D' is a vector space.
I.e. prove that it is closed under addition and scalar multiplication.

4 Ex 3 p 52 in Holden: Prove that for a regular distribution, $T_f = 0$ iff $f = 0$ a.e. ($f \in L^1_{loc}$).

5 Prove that $T_3 = \sum_{n=1}^{\infty} \delta_{\frac{1}{n}}$ belongs to $D'(0,1)$.
(Note that it does *not* belong to $D'(\mathbb{R})$).

6 Prove that $\partial^\alpha T \in D'$ for any $T \in D'$.
Hint: Verify that is it well-defined, linear, and continuous.

7 Prove that $T(\phi) = \sum_{n=1}^{\infty} \phi^{(n)}(n)$ defines a distribution on $\mathbb{R}$.

8 Holden Ex 5 p 52, first derivative only.