MA8105
Nonlinear PDEs and Sobolev spaces
Spring 2019

Exercise set 6

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$\boxed{1}$ Prove Hölder's inequality $\|fg\|_1 \leq \|f\|_p\|g\|_q$.

Hint: Exponential + Young, proof as for ℓ^p.

$\boxed{2}$ (Ex (4) p 93 in Holden) Prove Minkowski's inequality $\|f + g\|_p \leq \|f\|_p + \|g\|_p$.

Hint: $|f + g|^p \leq |f + g|^{p-1}(|f| + |g|)$ + Hölder.

$\boxed{3}$ (Ex (3) p 93 in Holden) Prove the generalized Hölder inequality.

Hint: We did the case of two functions in class, use this result and induction.

$\boxed{4}$ Prove $\|f * g\|_1 \leq \|f\|_1\|g\|_1$. Hint: Tonelli.

$\boxed{5}$ Prove Young's 2nd inequality $\|f(g * h)\|_1 \leq \|f\|_p\|g\|_q\|h\|_r$ using Hölder's inequality and Young's 1st inequality for convolutions.